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On the role of Λ in gravitational lensing

Summary

- How the lens equation is usually derived
- The contribution of Λ
- Observational effects

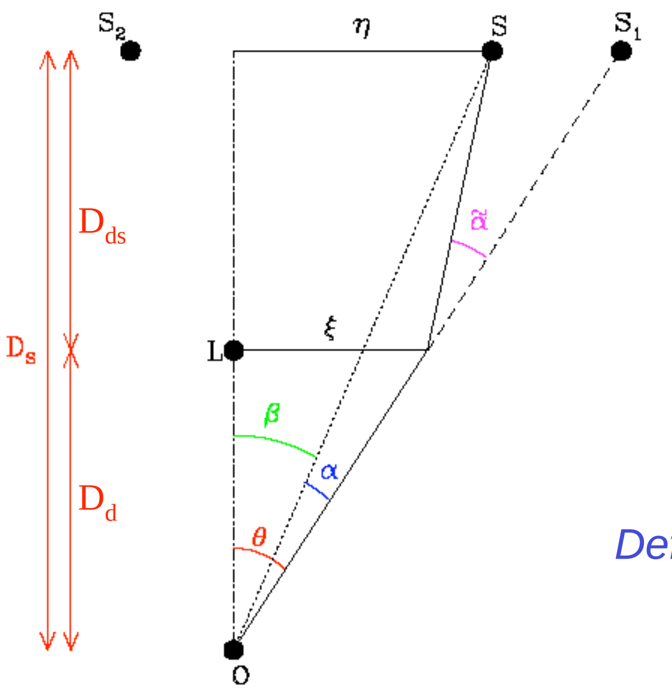
Sereno (2008) Phys. Rev. D 77, 043004
Sereno (2009) Phys. Rev. Lett. 102, 021301

But see also Islam; Kerr et al.; Rindler & Ishak; Schucker; Park; Khriplovich; Simpson, J.A. Peacock & A. F. Heavens; Gibbons et al.

Lens equation

Relation among:

- the true source position β
- the observed image position *θ*
- the deflection angle α, i.e. the difference of the initial and final ray directions



$$\beta = \theta - \alpha(\theta)$$

$$\alpha(\theta) = (D_{LS}/D_S)\tilde{\alpha}(\theta)$$

$$\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})$$

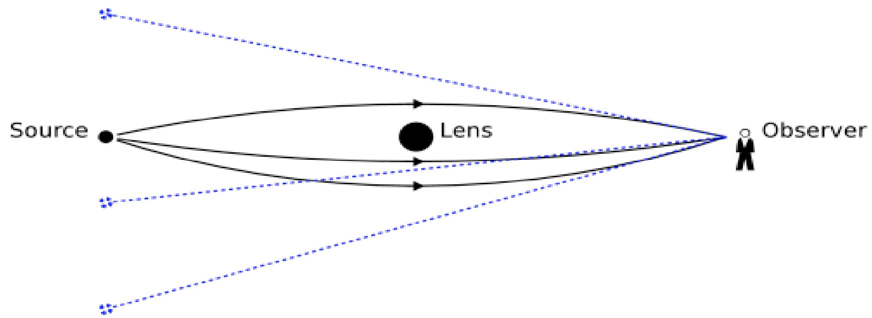
Multiple images occur when lens equation has multiple solutions

Deflection angle $$\vec{\tilde{\alpha}}(\vec{\xi}) = \frac{4G}{c^2} \int \frac{(\vec{\xi} - \vec{\xi}')\Sigma(\vec{\xi}')}{|\vec{\xi} - \vec{\xi}'|^2} d^2\xi'$$

Spherically symmetric lenses $$= \frac{4GM(\xi)}{c^2\xi}$$

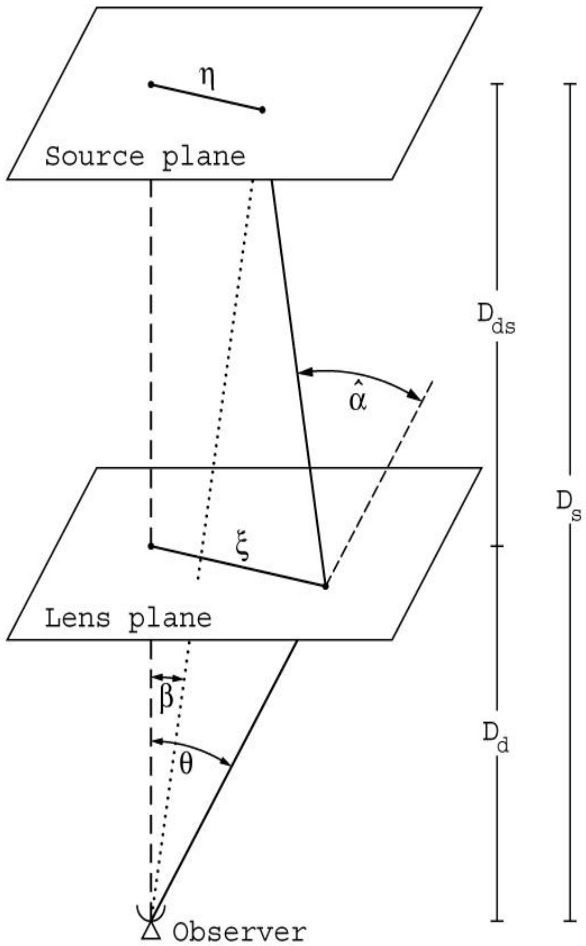
Gravitational lensing

Deflection of light by gravitational fields



1) Theory of gravity
2) metric for the space-time
3) Light trajectory in the coordinate space
4) Relation between coordinates and observables

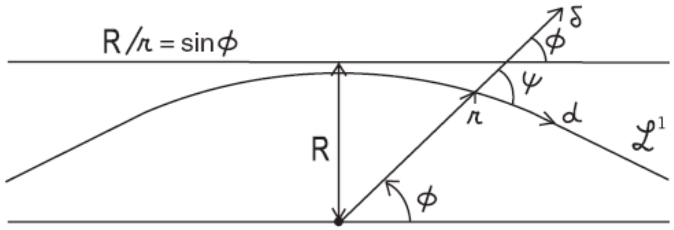
Framework for deflection



Lenses act as small, local perturbations in an otherwise spatially homogeneous and isotropic background.

Propagation in the smooth background, based either on FLRW (cosmological systems) or flat metrics (small systems)

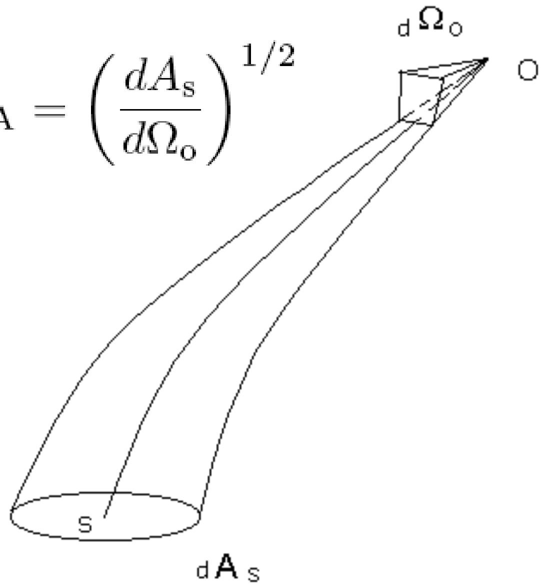
Deflection angle as a local effect, based on nearly Minkowskian, asymptotycally flat metrics. For a spherically symmetric lens



$$\hat{\alpha}(r_0) = 2\int_{r_0}^{\infty} \left| \frac{d\varphi}{dr} \right| dr - \pi$$

Background

Angular diameter distances characterize the background and the angle subtended by astronomical objects

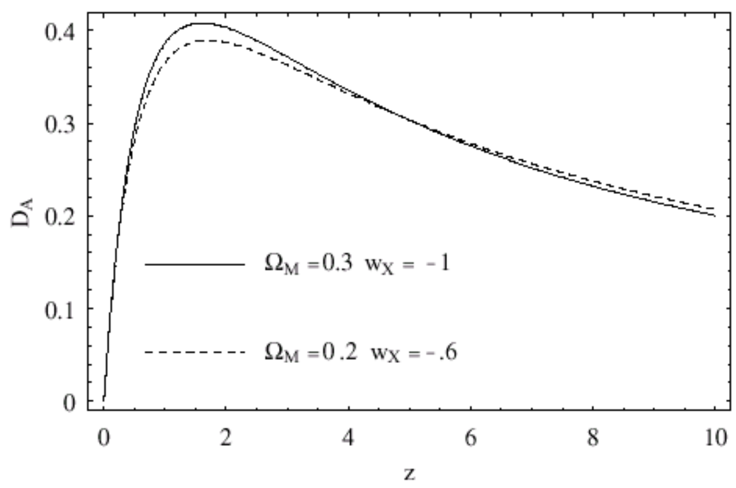


$$D_A = \left(\frac{dA_s}{d\Omega_o}\right)^{1/2}$$

In the Robertson-Walker metric (1936), $D \propto c/H_0 r(\Omega_M, \Omega_\Lambda, w_Q)$

$$D_A(z_s) = \frac{c}{H_0} \frac{1}{|\Omega_{K0}|^{\frac{1}{2}}(1+z_s)} \mathrm{Sinn}\left\{|\Omega_{K0}|^{\frac{1}{2}} \int_0^{z_s} \frac{H_0}{H(z)} dz\right\}$$

$$\frac{H(z)}{H_0} = \sqrt{\Omega_{M0}(1+z)^3 + \Omega_{\Lambda 0} + \Omega_{K0}(1+z)^2},$$



Dimensional analysis

Player	Kind	length scale	Size (Lensing cluster)	Deflection angle as a dimensionless combination (G=c=1)
Energy-Density				
Lens M	Local	$r_{\text{Sch}} \sim G\, M / c^2$	$M \sim 10^{15} M_{\text{sun}} \Longrightarrow 10^{18}\, m$	$M/b \sim 10^{-4}$ ~ 20 arcsec Higher orders ~ $(M/b)^n$
Cosmological constant Λ	*Local* *Global*	$r_{\Lambda} \sim (3/\Lambda)^{1/2}$	$\Lambda \sim 10^{-52}\ \text{m}^{-2} \Longrightarrow 10^{26}\, m$	$b\ m\ \Lambda \sim 10^{-12}$ < μarcsec $(D/\, r_{\Lambda})^2 \sim 10^{-2}$-1
Geometry				
Impact parameter b	Local		$b \sim 0.1\text{Mpc} \Longrightarrow 10^{22}\, m$	
Distances D_i	Global		$D \sim 1\text{Gpc} \Longrightarrow 10^{25}\, m$	

Gravitational lensing in the SdS

1) Theory of gravity	GR	$$R_{\mu\nu} - \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} S_{\mu\nu}$$
2) metric for the space-time	Schwarzschild-de Sitter	$$ds^2 = f_\Lambda(r)dt^2 - \frac{dr^2}{f_\Lambda(r)} - r^2\left(d\theta^2 - \sin^2\theta d\phi^2\right),$$ $$f_\Lambda(r) \equiv \left(1 - \frac{2m}{r} - \frac{\Lambda r^2}{3}\right)$$ + accounts for local mass and global expansion
3) Light trajectory in the coordinate space	Null Geodesic	$$\phi_s = \pm \int \frac{dr}{r^2}\left[\frac{1}{b^2} + \frac{1}{r_\Lambda^2} - \frac{1}{r^2} + \frac{2m}{r^3}\right]^{-1/2}$$
4) Relation between coordinates and observables		$$\sin\vartheta = \frac{\sqrt{1 - v^{[r]2}(r_o)}}{1 - v^{[r]}(r_o)\sqrt{1-(b/r_o)^2 f_\Lambda(r_o)}} \frac{b}{r_o}\sqrt{f_\Lambda(r_o)},$$

Old paradigm

Global effect: Λ affects light propagation in the background, far from the lens

Near the lens. No contriution to the deflection angle (Islam 1986). Λ does not enter in the orbital equations of the SdS metric (space-time with a central mas and a cosmological constant)

$$\frac{d^2u}{d\phi^2} + u = 3mu^2, \qquad (u \equiv 1/r),$$

What is missing: In the observer's frame, Λ affects the actual observations that can be made on the orbit equation (Rindler & Ishak 2007)

Geodesic motion

Light path from the observer to the source. Due to spherical symmetry, motion is confined to a plane $\{r_o, \phi_o = 0\}$ $\{r_s, \phi_s\}$

$$\phi_s = \pm \int \frac{dr}{r^2}\left[\frac{1}{b^2} + \frac{1}{r_\Lambda^2} - \frac{1}{r^2} + \frac{2m}{r^3}\right]^{-1/2} = \pi - \frac{4m}{b} + b\left(\frac{1}{r_s} + \frac{1}{r_o}\right) + \frac{15m^2\pi}{4b^2} + \frac{128m^3}{3b^3} + \frac{b^3}{6}\left(\frac{1}{r_s^3} + \frac{1}{r_o^3}\right) + \frac{3465m^4\pi}{64b^4} + \frac{3584m^5}{5b^5} + \frac{2mb}{r_\Lambda^2} - \frac{mb^3}{4}\left(\frac{1}{r_s^4} + \frac{1}{r_o^4}\right) + \frac{3b^5}{40}\left(\frac{1}{r_s^5} + \frac{1}{r_o^5}\right) - \frac{b^3}{2r_\Lambda^2}\left(\frac{1}{r_s} + \frac{1}{r_o}\right) + \mathcal{O}(\epsilon^6).$$

Constant of motion (impact parameter) $b(\equiv \dot{\phi} r^2)$

0-th order equation

Higher order Schwarzschild corrections to α

Geometrical corrections

Λ coupling to the mass

Weak deflection limit. *Light path far from the lens*

Two expansion parameters

$$m/b \equiv \epsilon_m \ll 1$$

$$\epsilon_\Lambda \equiv b/r_\Lambda$$

$$b/r_o \sim b/r_s \sim \epsilon_m$$

$$r_o \sim r_s \lesssim r_\Lambda$$

Image position

Einstein ring, scaled units

$$\vartheta_E \equiv \sqrt{4mD/D_d} \qquad D \equiv D_{ds}/D_s \qquad \varepsilon_{\mathrm{E}} \equiv \vartheta_{\mathrm{E}}/4D$$

Critical lines

Λ effect

$$\vartheta_t \simeq \vartheta_t(\Lambda=0) + \left\{ \frac{1}{4}\left(1-\frac{1}{2D}\right)\frac{1}{r_{\Lambda\varepsilon}^2} + \frac{1}{4r_{\Lambda\varepsilon}(1-2Dr_{\Lambda\varepsilon})}\left(1+\frac{1-4Dr_{\Lambda\varepsilon}}{2D}\delta v^{(2)}\right)\right\}\varepsilon^2$$

Observer motion

$$v^{[r]} \simeq (r_o/r_\Lambda)(1+\delta v^{(2)}\varepsilon^2)$$

- ❖Static or Moving observers
  - Comoving or with peculiar velocities
- ❖Background distances defined in the associated RW space-time in a number of ways

- ❖ Some oberver-dependent terms
- ❖ Local contribution due to local bending

$$\delta\vartheta_t^\Lambda = (1/4)(D_d/r_\Lambda)^2\vartheta_E^3$$

Lens equation

In a generic ΛCDM universe, we pick up only the local Λ contribution to the bending

$$\delta\hat{\alpha}_\Lambda = \frac{\vartheta_E^2}{2}\left(\frac{D_d}{r_\Lambda}\right)^2 \vartheta \qquad \Longrightarrow \qquad \beta \simeq \vartheta - \frac{D_{ds}}{D_s}(\hat{\alpha} + \delta\hat{\alpha}_\Lambda)$$

Now distances have to account for homogeneous and clumped dark matter

Two effects from cosmological constant:

- Attractive contribution from coupling effect
- The repulsive gravitational effect of Λ in the background counteracts deflection

Unperturbed images

$$\vartheta_0 = (\beta \pm \sqrt{\beta^2 + 4\vartheta_E^2})/2$$

Perturbed images

$$\vartheta \simeq \vartheta_0\left\{1 + \frac{D_d^2}{2r_\Lambda^2}\frac{\vartheta_0^2}{1 + \vartheta_0^2/\vartheta_E^2}\right\}$$

Cluster of galaxies: $M \sim 10^{15}\, M_{Sun}$ $z_d \sim 0.3$, $z_s \sim 1$ $\Longrightarrow \alpha_\Lambda \sim 0.1\,\mu$arcsec

Near Lenses

- Λ effect is $\sim (r_0 / r_\Lambda)^2$ smaller than the main term
- $r_\Lambda \sim 5000$ Mpc

Solar light deflection

• deviations from GR predictions within 0.05% $\Longrightarrow \Lambda \leq 10^{-25}\ \mathrm{m}^{-2}$ (17 odg worse than limits from precession shift)

Sgr A*

• ($d \sim 7.6$ kpc, $M \sim 3.6 * 10^6\ M_{sun}$) $\Longrightarrow \delta\vartheta_\Lambda \sim 10^{-14}$ arcsec

M31 microloensing

• ($d \sim 750$ kpc) optical depth change $\Longrightarrow 2\delta\vartheta_E/\vartheta_E \sim 10^{-8}$

Redshift difference

Due to the cosmological constant, the observed source redshift depends on the impact parameter of the ligh trajectory. Λ brings about a difference in the redshift of multiple images

General definition $$1 + z_s = g_{\alpha\beta} k_s^\alpha U_s^\beta / g_{\alpha\beta} k_o^\alpha U_o^\beta$$

In terms of the observed image positions

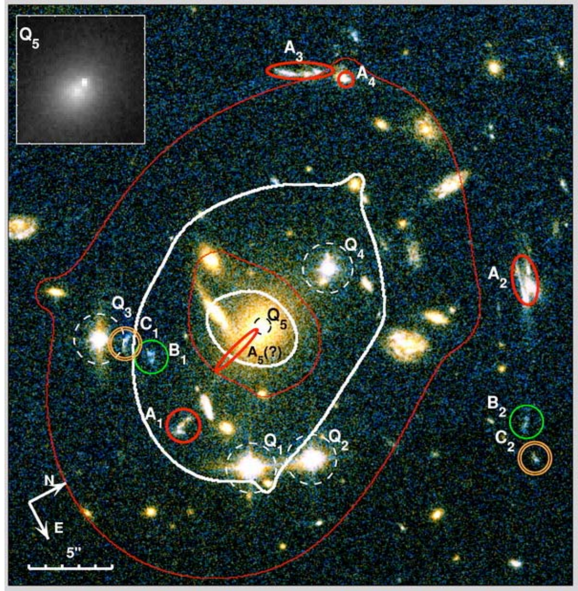
$$\Delta z_s^{\mathrm{Obs}} \simeq \frac{1+z_s}{1+z_d}\left(\frac{1}{z_d} - \frac{1}{z_s}\right)^{-1} \frac{\vartheta_+^2 - \vartheta_-^2}{2} \qquad \vartheta_+^2 - \vartheta_-^2 \simeq \beta\sqrt{\beta^2 + 4\vartheta_E^2}$$

z_d, z_s are the redshifts in the absence of the lens, as measured in the associated spacetime

The shift is $\sim\vartheta_E^2$

Cluster of galaxies: $M\sim10^{15}\,M_{Sun}$ $z_d\sim 0.3$, $z_s\sim1$ $\Longrightarrow \Delta z_s\sim10^{-7}$

Observational prospects



Lensed quasars

• Quasars lensed by galaxy clusters have already been observed in SDSS J1004+411 (ACS image from Sharon et al. 2005)

Deflection angle

• Forthcoming Gaia mission will produce in five years a full-sky map of roughly 500 000 quasars; by making of order 100 repeated measurements over the 5 yr mission, Gaia will hopefully achieve a positional error between 10 and 200 micro-arcsec (for quasars with magnitude V=15-20).

Redshift split

• An high resolution spetrograph like proposed CODEX @E-ELT could measure redshist with a resolution as good as few cm/s ($z \sim 10^{-9}$)

Conclusions

➢ Λ affects lensing in a peculiar way. Other dark energy proposal are expected to act differently

✓Tool for breaking degeneracies

➢Novel effects with sizes on the edge of measurability