Effect of peculiar motion in weak lensing

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Overview

- Description of the relativistic calculation of the shear γ and the convergence κ, in order
  - to recover the standard formula
  - to see the effect of peculiar velocities

C. Bonvin, PRD 78, 123530 (2008)

- The effect on the shear is second order. Consequently, it is negligible in the calculation of the power spectrum.
- The effect on the convergence is first order, hence peculiar velocities have an observable impact on the power spectrum.
- The consistency relation between κ and γ is modified by peculiar motion $\Rightarrow$ we could use this in order to measure the peculiar velocities of galaxies.

Weak lensing

Lensing describes the deflection of light by the gravitational potential $\Rightarrow$ it is sensitive to the distribution of matter.

The distortion created by weak lensing can be split in two parts: the convergence and the shear.

Observation of the shear

The correlation between the ellipticity of galaxies is measured.

First measurements in 2000 : Bacon *et al.*, Kaiser *et al.*, Wittman *et al.*, L. van Waerbeke *et al.*. Recently: Fu *et al.* (2007), with CFHTLS.

Future measurements (DES, Pan-STARR, EUCLID, LSST...) should reach 1% accuracy.

Observation of the convergence

The intrinsic size of the galaxies is unknown.

However, the number of galaxies per unit of solid angle at redshift z and flux f is known. The convergence modifies this number, by modifying the flux and the size of the solid angle.

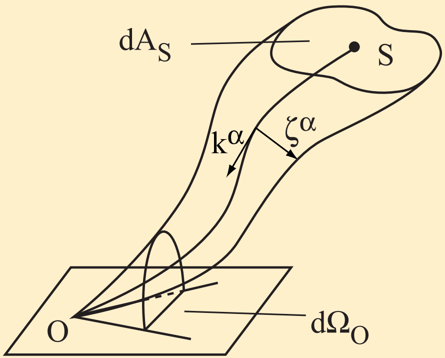
Recent measurements, Ménard *et al.* (2009), Hildebrandt *et al.*, (2009) are in agreement with shear observations.

Challenge: get rid of intrinsic clustering by

- correlating objects widely separated in redshift
- removing close pairs of galaxies from the signal

In the future, the SKA plan to measure the convergence with 1% accuracy, using the second method Zhang and Pen, PRL (2005)

The Jacobi map



k^α photon direction
ξ^α connection vector

Sachs (1961)

$$\frac{D^2\xi^\alpha(\lambda)}{D\lambda^2} = R^\alpha_{\beta\gamma\delta} k^\beta k^\gamma \xi^\delta$$

Solution: $\xi^\alpha_S = J^\alpha_\beta(S,O)\delta\theta^\beta_O$

- ξ^α_S describes the surface of the source.
- $\delta\theta^\alpha_O$ describes the angle of observation.
- J^α_β, the Jacobi map, relates the galaxy to its image.

J^α_β is a 4x4 matrix. How do we relate it to κ and γ?

The Jacobi map

Usually, the 4x4 matrix is reduced to a 2x2 submatrix.

The galaxy and the image are 2-dimensional $\Rightarrow$ we can project $J^{\alpha}_{\ \beta}$ on the appropriate basis to extract

$$D_{ab} = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$$

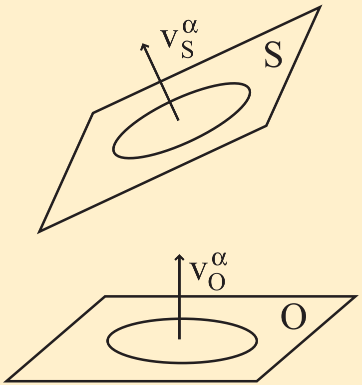
Trace	$\longrightarrow$	convergence κ
Traceless part	$\longrightarrow$	shear γ_1, γ_2

$$\kappa = (E_1^i E_1^j + E_2^i E_2^j)\partial_i\partial_j \hat{\Psi}_S \equiv \Delta_\perp \hat{\Psi}_S$$

$$\gamma_1 = (E_1^i E_1^j - E_2^i E_2^j)\partial_i\partial_j \hat{\Psi}_S \qquad \gamma_2 = 2E_1^i E_2^j \partial_i\partial_j \hat{\Psi}_S$$

Integrated potential: $\displaystyle \hat{\Psi}_S = \int_{\eta_S}^{\eta_O} d\eta \frac{(\eta-\eta_S)(\eta_O-\eta)}{\eta_O-\eta_S}\Psi(\eta, \mathbf{x}(\eta))$

First effect of the velocity



We can not reduce the 4x4 Jacobi map to a 2x2 matrix.

Due to peculiar velocities, the plane of the galaxy is not parallel to the plane of the image.

The plane of the galaxy is orthogonal to v_S^α, whereas the plane of the image is orthogonal to v_O^α.

In an inhomogeneous Universe, we have $v_S^\alpha \neq v_O^\alpha$.

$\Rightarrow$ We need a 3x3 matrix to relate the galaxy to its image.

Shear deformation

We apply the 3x3 matrix on a circular source of radius r.

The image (θ_1, θ_2) obeys

$$\frac{\theta_1^2}{a^2} + \frac{\theta_2^2}{b^2} = 1$$

$$a = r \qquad\qquad b = \frac{r}{\sqrt{1 + \left[(\mathbf{v}_S - \mathbf{v}_O)\mathbf{E}_1\right]^2}}$$

The shear deformation is quadratic in the velocity difference.

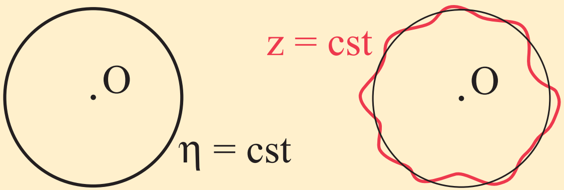
$\Rightarrow$ It is negligible in the calculation of the power spectrum, but it will play a role in the calculation of the bispectrum.

Second effect of the velocity

Effect on the redshift z, that affects the convergence.

Usually, $\kappa(\mathbf{n}, \eta_S), \gamma(\mathbf{n}, \eta_S)$ are calculated. However, η_S is not an observable quantity. We need to express κ and γ as functions of the redshift z_S.

$$1 + z_S = \frac{a_O}{a_S}\left(1 + \Psi_S - \Psi_O + 2\int_{\eta_S}^{\eta_O} d\eta\ \mathbf{n}\cdot\nabla\Psi + (\mathbf{v}_O - \mathbf{v}_S)\cdot\mathbf{n}\right)$$



We measure correlations on a distorted sphere $\Rightarrow$

New contribution to the convergence.

Amplitude of the effect

$$\kappa_\Psi = \int_{\eta_S}^{\eta_O} d\eta \frac{(\eta - \eta_S)(\eta_O - \eta)}{\eta_O - \eta_S} \Delta_\perp \Psi$$

$$\kappa_{\mathbf{v}} = \left(1 - \frac{1 + z_S}{H_S(\eta_O - \eta_S)}\right) (\mathbf{v}_S - \mathbf{v}_O) \cdot \mathbf{n}$$

The observable quantity associated with the convergence is the overdensity of galaxies

Broadhurst, Taylor and Peacock (1995)

$$\delta_g = \frac{n(f) - n_0(f)}{n_0(f)} \simeq 2\big(\alpha - 1\big)\left(\kappa_\Psi + \kappa_{\mathbf{v}}\right)$$

α is related to the modelization of $n_0(f)$: it is known.

The angular power spectra

We calculate the two-points correlations and we determine the angular power spectrum

$$\langle \delta_g(z_S, \mathbf{n}) \delta_g(z_S, \mathbf{n}') \rangle = \sum_\ell \frac{2\ell + 1}{4\pi} C_\ell(z_S) P_\ell(\mathbf{n} \cdot \mathbf{n}')$$

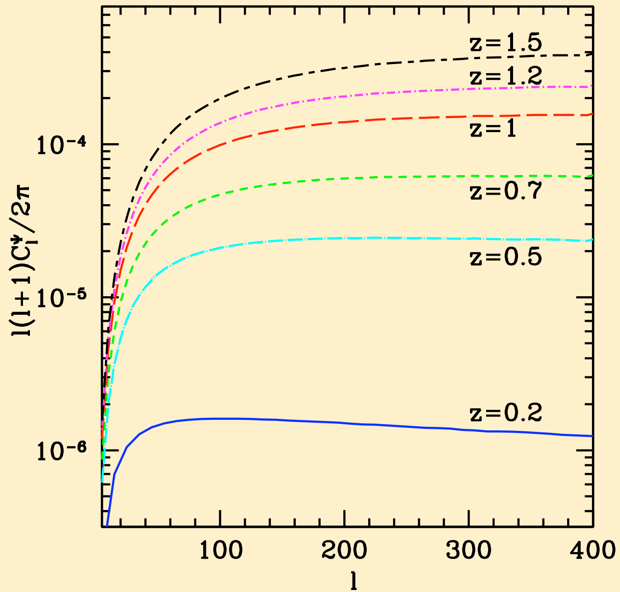
The angular power spectrum contains two contributions

$$C_\ell = C_\ell^\Psi + C_\ell^{\mathbf{v}}$$

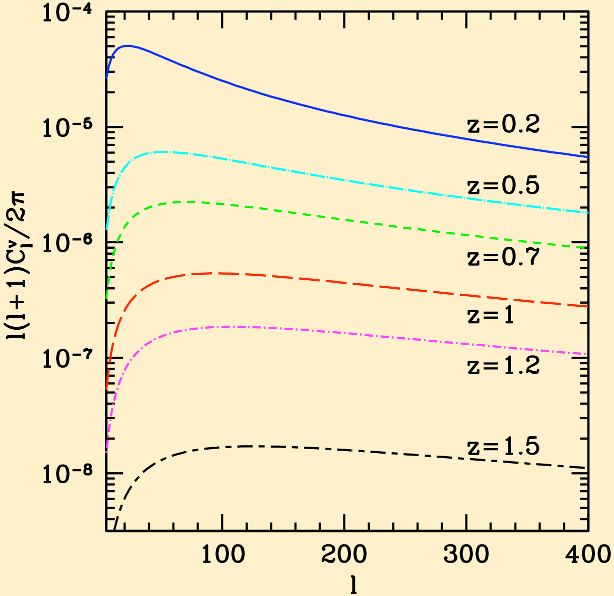
Question: is $C_\ell^{\mathbf{v}}$ large enough to be detected?

We use Einstein's equations to relate $\mathbf{v}$ to Ψ, and we choose a gaussian primordial power spectrum for Ψ.

The angular power spectra

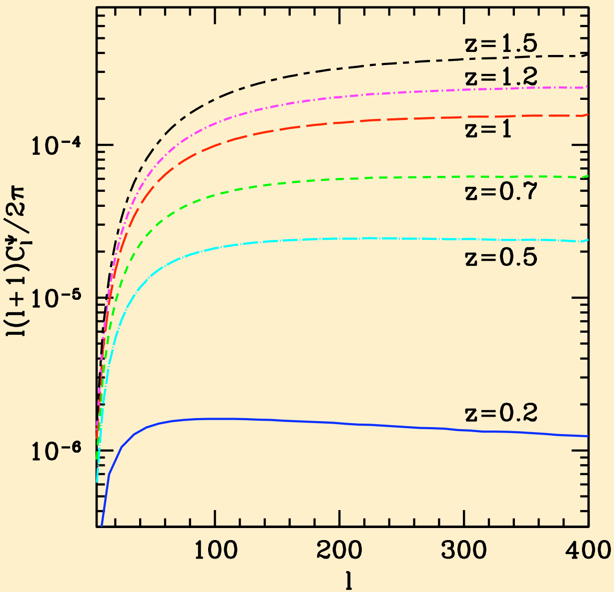


Contribution from the potential κ_Ψ

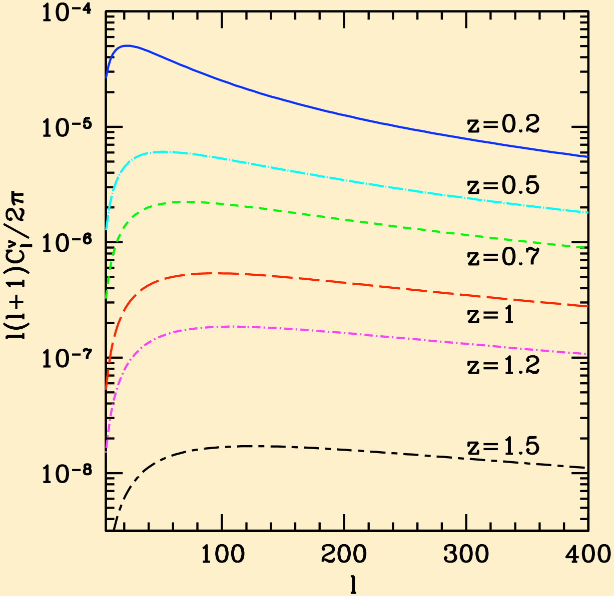


Contribution from the velocity $\kappa_{\mathbf{v}}$

The angular power spectra OBSERVABLE



Contribution from the potential κ_Ψ



Contribution from the velocity $\kappa_{\mathbf{v}}$

Consistency relation

The standard contributions of the shear and the convergence are not independent.
The angular power spectra satisfy

$$\frac{\ell(\ell+1)}{(\ell+2)(\ell-1)}C_\ell^\gamma = C_\ell^{\kappa_\Psi} \qquad \text{Hu (2000)}$$

Peculiar velocities affect the convergence at first order, but not the shear. Therefore the consistency relation becomes

$$\frac{\ell(\ell+1)}{(\ell+2)(\ell-1)}C_\ell^\gamma = C_\ell^{\kappa_{tot}} - C_\ell^{\kappa_\mathbf{v}}$$

We can use this relation to measure the peculiar velocities of galaxies.

Conclusion

- The peculiar velocity of galaxies affect weak lensing observations.

- The effect on the convergence power spectrum is large enough, at $z \leq 1$, to be detected by future experiments.

- The consistency relation between the shear and the convergence is modified by peculiar velocities.

  $\longrightarrow$ We could use this to measure the velocities of galaxies.

- In future calculations, it could be interesting to include non-linear velocities, especially at small redshifts. This could increase the signal significantly.

Characteristic of experiment

In order to detect the effect of peculiar motion, an experiment must satisfy 3 criteria

- cover a large part of the sky: $C_\ell^{\mathbf{v}}$ pics at $\ell \sim 30 - 150$, i.e. $\theta \sim 70$ arcmin -6 degrees.

  CFHTLS already up to 4 degrees, near future 8 degrees. EUCLID, SKA 20'000 square degrees.

- remove intrinsic clustering $\longrightarrow$ precise measurements of z

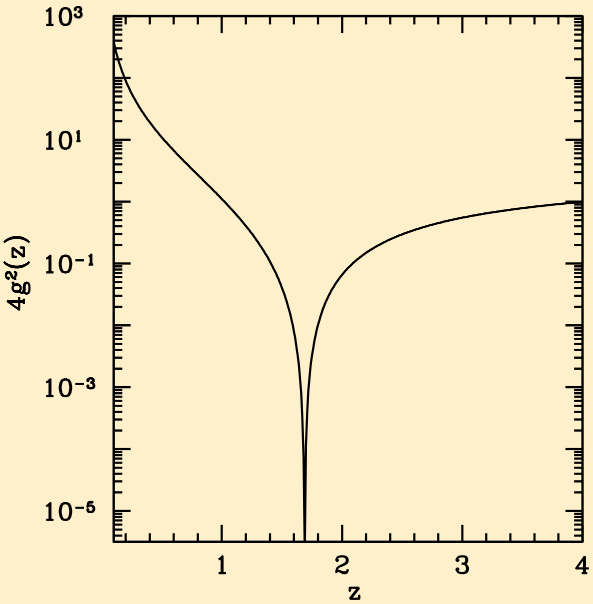
  SKA: 21cm emission line

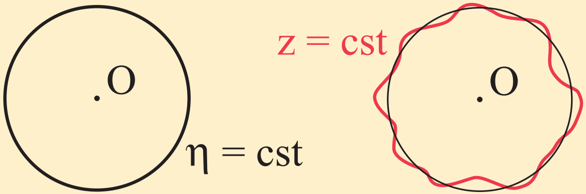
  other: photometric measurements

- cover redshifts smaller than 1.

Evolution of the velocity term

$$\kappa_{\mathbf{v}} = \left(1 - \frac{1}{\mathcal{H}_S(\eta_O - \eta_S)}\right)(\mathbf{v}_S - \mathbf{v}_O) \cdot \mathbf{n}$$



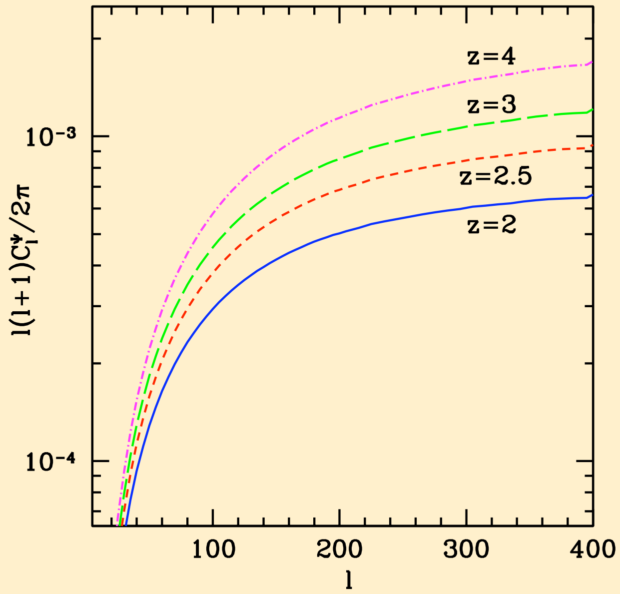


Two opposite effects:

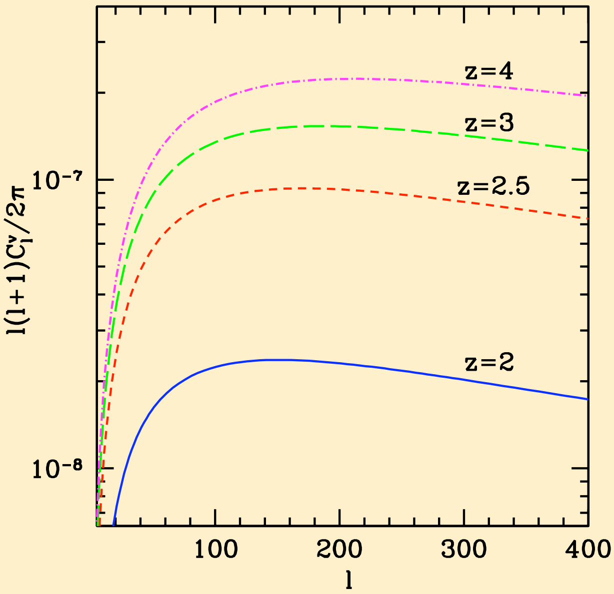
Further away → smaller angle → demagnification

Smaller scale factor → larger stretch → magnification

The angular power spectra



Contribution from the potential κ_Ψ



Contribution from the velocity $\kappa_\mathbf{v}$