Invisible Universe 2009 in Paris, 07/01

Geometrical Inflation in Higher-derivative Supergravity

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- Motivation: modified supergravity for inflation and quintessence
- Possible origin of modified supergravity from superstrings
- Modified supergravity from M-theory
- Generalized Friedmann equations, exact solutions for inflation
- Holomorphic $F(\mathcal{R})$ supergravity
- Scalar potential in $F(\mathcal{R})$ supergravity
- Conclusion and Discussion

Adv. High Energy Phys: 521389 (2008), Int. Journ. Mod. Phys. A23 (2008) 153,
Phys. Lett. B684 (2009) 207, Classical and Quantum Gravity 26 (2009)135006

General Motivation

Homogeneity and isotropy of our Universe, as well as its observed spectrum of density perturbations (CMB), are well explained by inflationary cosmology. Inflation also solves the horizon, flatness, monopole production and initial conditions problems. However, inflation remains to be a paradigm, not (yet) a theory

- Inflation and quintessence are usually realised by introducing *ad hoc* a scalar field (inflaton) and the appropriate scalar potential. When using Einstein equations, it gives rise to a massive violation of the strong energy condition, and exotic matter with large negative pressure (dark energy), respectively. Despite of the apparent simplicity of those scenarios, the origin of their key ingredients remains obscure
- There is no shortage of inflationary models and models of dark energy in modern cosmology, partly due to

(i) large variety of theoretical tools (cosmological constant, modified gravity, extra dimensions, strings and branes, etc.),
(ii) our poor understanding of gravity and especially, of quantum gravity,
(iii) experimental limits on precision measurements outside our solar system

Unification of Standard Models of EP and Cosmology

- Though most of facts about Elementary Particles (EP) and our Universe are well described by the Standard Models (SMs) of High-Energy Physics and Cosmology, respectively, there is little doubt that both have to be modified and, ultimately, united and unified. Theoretical solutions beyond the SMs, both in HEP and in Cosmology, ideally should come in one package, within a single fundamental theory of ALL fundamental physical interactions, including quantum gravity
- A lot of new experimental data expected soon, both in HEP (LHC), and from the skies (PLANCK probe of CMB, X-ray eROSITA, ESA's Euclid, NASA's JDEM)
- In part, expected supersymmetry beyond the SM of EP motivates the use of SUPERGRAVITY beyond the SM of Cosmology

Note: MSSM neutralinos are natural candidates for a stable (WIMP) Dark Matter!

- Any fundamantal theory of quantum gravity gives rise to a modified (higher-derivative) gravity, as its effective field-theoretical description. So we propose a higher-derivative supergravity (with spontaneously broken supersymmetry) as the effective tool of describing inflation and quintessence, and as a supersymmetric generalization of modified gravity

Modified Supergravity and Superstrings

- Theory of superstrings is the leading candidate for a unified fundamental theory, it offers a consistent perturbative quantum gravity, and it is capable to generate the SM of EP as well as the cosmological SM
- Many (ad hoc) inflationary scenarios using branes (string theory solitons), were proposed, together with their embeddings into (warped) compactified superstrings (KKLT, etc.). A number of possibilities is huge, about 10^{500}, known as the String Landscape (hence, no predictive power)

- A fully non-perturbative superstring theory (or M-theory) is not known. The perturbative superstrings are merely defined on-shell (in the form of quantum amplitudes), while they give rise to the infinitely many higher order curvature corrections to Einstein equations, to all orders in Regge slope parameter α' and string coupling g_s (in 10 space-time dimensions). Finite (off-shell) form of those corrections (needed for cosmology) is unknown and is beyond control at present.

Our modified supergravity in (1+3)-spacetime dimensions

- We consider only geometrical (gravitational) terms in the four-dimensional superstring effective action, in our search for the universal gravity-driven mechanisms of the early Universe inflation and the present Universe acceleration (due to quantum gravity physics). We assume that quantum g_s-corrections are suppressed against leading α'-corrections, whereas most of the moduli (except, perhaps, dilaton and axion) are stabilized (e.g. by fluxes, after the warped compactification to four spacetime dimensions and spontaneous supersymmetry breaking)

- In fact, our four-dimensional framework does not really depend much upon the input of superstrings (we use the latter as a guide mostly). So our results are pretty generic and may even be relevant to any quantum gravity theory

- The vacuum energy density $\rho_{\rm inf}$ during inflation is bounded from above by a non-observation of tensor fluctuations of CMB radiation (WMAP 2007),

$$\rho_{\rm inf} \leq \left(10^{-3} M_{\rm Pl}\right)^4 \qquad (1)$$

Effective higher-derivative supergravity from M-theory

- M-theory is non-perturbative upgrade of superstrings (Witten, 1995)

(i) M-theory low-energy effective action is unique=supergravity in 11 dimensions,
(ii) the next quantum gravitational corrections to the 11-dimensional supergravity from M-theory are all quartic in the curvature.

The quartic bosonic terms of the M-theory corrected 11-dim supergravity are

$$S_{11} = -\frac{1}{2\kappa_{11}^2}\int d^{11}x\sqrt{-g}\left[R - \frac{1}{2\cdot 4!}F^2 - \frac{1}{6\cdot 3!\cdot (4!)^2}\varepsilon_{11}CFF\right]$$
$$-\frac{T_2}{(2\pi)^4\cdot 3^2\cdot 2^{13}}\int d^{11}x\sqrt{-g}\left(J - \frac{1}{2}E_8\right) + T_2\int C\wedge X_8 \qquad (2)$$

where κ_{11} is the gravitational constant (in 11 dims), T_2 is the M2-brane tension,

$$T_2 = \left(\frac{2\pi^2}{\kappa_{11}^2}\right)^{1/3} \qquad (3)$$

Our notation in 11 (spacetime) dimensions

C is a 3-form gauge field of the 11-dimensional supergravity, and $F = dC$ is its four-form field strength, R is the gravitational scalar curvature, ε_{11} stands for the 11-dimensional Levi-Civita symbol in the Chern-Simons-like coupling, while (J, E_8, X_8) are certain quartic polynomials in the 11-dimensional curvature:

$$J = 3 \cdot 2^8 \left(R^{mijn} R_{pijq} R_m{}^{rsp} R^q{}_{rsn} + \frac{1}{2} R^{mnij} R_{pqij} R_m{}^{rsp} R^q{}_{rsn} \right) + \mathcal{O}(R_{mn}) \tag{4}$$

the E_8 is the 11-dimensional extension of the 8-dimensional Euler density,

$$E_8 = \frac{1}{3!} \varepsilon^{abcm_1n_1 \ldots m_4n_4} \varepsilon_{abcm'_1n'_1 \ldots m'_4n'_4} R^{m'_1n'_1}{}_{m_1n_1} \cdots R^{m'_4n'_4}{}_{m_4n_4} \tag{5}$$

and the X_8 is the 8-form

$$X_8 = \frac{1}{192 \cdot (2\pi^2)^4} \left[\mathrm{tr} R^4 - \frac{1}{4} (\mathrm{tr} R^2)^2 \right] \tag{6}$$

- The J-contribution (4) is defined modulo Ricci-dependent terms by its very derivation (on-shell) !

Warped compactification to 4 spacetime dimensions

- To match the constraints imposed by particle physics, M-theory should be compactified on a 7-dimensional space of special (G_2) holonomy (Atiyah, Witten)
- In the presence of fluxes, we consider a warped compactification

$$ds_{11}^2 = e^{2A(y)} ds_{\mathsf{FRW}}^2 + e^{-2A(y)} ds_7^2 \tag{7}$$

where $ds_{\mathsf{FRW}}^2(x)$ is the (uncompactified) 4-dim FRW metric, $ds_7^2(y)$ is a (compactified) 7-dim metric, and $A(y)$ is a warp factor, $\nabla^2 A \propto \ (\mathsf{fluxes})^2$.

- Since we are interested in the purely gravitational terms in 4 dims, the explicit form of the 7-metric ds_7^2 is not needed. After dimensional reduction, the only gravitational terms in 4 dims are

$$S_4 = -\frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \, (R + \beta J_R) \tag{8}$$

where we have introduced the Einstein coupling κ in four dimensions, and the four-dimensional counterpart J_R of J in eq. (4), $i, j = 0, 1, 2, 3$,

$$J_R = R^{mijn} R_{pijq} R_m{}^{rsp} R^q{}_{rsn} + \frac{1}{2} R^{mnij} R_{pqij} R_m{}^{rsp} R^q{}_{rsn} + O(R_{mn}) \tag{9}$$

Couplings and scales

The relation between the coupling constants κ_{11} and κ is $\kappa^2 = e^{5A} M_{\mathsf{KK}}^7 \kappa_{11}^2$ where the Kaluza-Klein (KK) compactification scale is $M_{\mathsf{KK}}^{-7} = Vol_7 \equiv \int d^7y \sqrt{g_7}$, and the average warp factor A (of integer weight p) is defined by

$$e^{pA} = \frac{1}{Vol_7} \int d^7y \sqrt{g_7}\, e^{pA(y)} \tag{10}$$

It follows

$$\beta = \frac{1}{3} \left(\frac{\kappa^2}{2^{23/2} \pi^5 e^{14A} M_{\mathsf{KK}}^7} \right)^{2/3} \tag{11}$$

of mass dimension -6. When using the Planck scale $\kappa \approx 10^{-33}$cm and $M_{\mathsf{KK}}^{-1} \approx 10^{-15}$cm, and ignoring the warp factor, $A = 0$, we get (in fact, unacceptable) value $\beta \approx 10^{-118}\,\mathrm{cm}^6$. Altogether it leads to the modified gravitational equations

$$R_{ij} - \tfrac{1}{2} g_{ij} R + \beta \frac{1}{\sqrt{-g}} \frac{\delta}{\delta g^{ij}} \left(\sqrt{-g} J_R \right) = \kappa^2 T_{ij} \tag{12}$$

where T_{ij} stands for the energy-momentum tensor of all matter fields (including dilaton and axion)

On-shell structure of the quartic gravity terms

Detailed structure and physical meaning of the quartic curvature terms are revealed via their connection to the four-dimensional Bel-Robinson (BR) tensor

$$T_R^{iklm} = R^{ipql} R^k{}_{pq}{}^m + {}^*R^{ipql\,*}R^k{}_{pq}{}^m \tag{13}$$

whose structure is quite similar to that of the Maxwell stress-energy tensor, *cf.*

$$T_{ij}^{\text{Maxwell}} = F_{ik}F_j{}^k + {}^*F_{ik}{}^*F_j{}^k \quad , \quad F_{ij} = \partial_i A_j - \partial_j A_i \tag{14}$$

Weyl cousin T_C^{ijlm} of the BR tensor is obtained by replacing all curvatures by Weyl tensors. It is factorized in the 2-component (spinor) formalism,

$$(T_C)_{\alpha\beta\gamma\delta\dot\alpha\dot\beta\dot\gamma\dot\delta} = C_{\alpha\beta\gamma\delta}\bar{C}_{\dot\alpha\dot\beta\dot\gamma\dot\delta} \tag{15}$$

Important! : here we consider all the quartic terms on-shell, i.e. modulo Ricci-tensor-dependent terms. Then we find

$$T^2_{ijkl} = 8J_R = \tfrac{1}{4}(R_{ijkl}R^{ijkl})^2 + \tfrac{1}{4}({}^*R_{ijkl}R^{ijkl})^2 \tag{16}$$

and

$$T^2_{ijkl} = 8J_R = -\tfrac{1}{4}({}^*R^2_{ijkl})^2 + \tfrac{1}{4}({}^*R_{ijkl}R^{ijkl})^2$$
$$= \tfrac{1}{4}(P_4^2 - E_4^2) = \tfrac{1}{4}(P_4 + E_4)(P_4 - E_4) \qquad (17)$$

in terms of the Euler and Pontryagin topological densities in four dimensions.

The on-shell BR tensor is fully symmetric and traceless,

$$T_{ijkl} = T_{(ijkl)} \ , \qquad T^i_{ikl} = 0 \qquad (18)$$

is covariantly conserved

$$\nabla^i T_{ijkl} = 0 \qquad (19)$$

and has a positive 'energy' density,

$$T_{0000} > 0 \qquad (20)$$

When using *Riemann Normal Coordinates* (RNC) at a given spacetime point, one has

$$g_{ij} = \eta_{ij} \ , \quad g_{ij,k} = 0 \ , \quad g_{ij,mn} = -\tfrac{1}{3}(R_{imjn} + R_{injm}) \qquad (21)$$

and

$$\Gamma^{i}_{jk,l} = -\tfrac{1}{3}(R^{i}{}_{jkl} + R^{i}{}_{kjl}) \tag{22}$$

Raising and lowering of vector indices are done with Minkowski metric η_{ij} and its inverse η^{ij}, so that all traces in the last two eqs. (21) and (22) vanish.

BR tensor is simply related to the gravitational energy-momentum pseudo-tensors in RNC (Deser):

$$T_{ijkl} = \partial_k \partial_l \left(t^{LL}_{ij} + \tfrac{1}{2} t^{E}_{ij} \right) \tag{23}$$

where the symmetric Landau-Lifshitz gravitational pseudo-tensor is

$$\begin{aligned} (t_{LL})^{ij} = & -\eta^{ip}\eta^{jq}\Gamma^{k}_{pm}\Gamma^{m}_{qk} + \Gamma^{i}_{mn}\Gamma^{j}_{pq}\eta^{mp}\eta^{nq} \\ & - \left(\Gamma^{m}_{np}\Gamma^{j}_{mq}\eta^{in}\eta^{pq} + \Gamma^{m}_{np}\Gamma^{i}_{mq}\eta^{jn}\eta^{pq} \right) + h^{ij}\Gamma^{m}_{np}\Gamma^{n}_{mq}\eta^{pq} \end{aligned} \tag{24}$$

and the non-symmetric Einstein gravitational pseudo-tensor is

$$(t^{E})^{i}_{j} = \left(-2\Gamma^{i}_{mp}\Gamma^{m}_{jq} + \delta^{i}_{j}\Gamma^{n}_{pm}\Gamma^{m}_{qn} \right) \eta^{pq} \tag{25}$$

FRW metric and inflation

- The main Cosmological Principle of a spatially homogeneous and isotropic $(1+3)$-dimensional universe (at large scales) gives rise to the FRW metric

$$ds^2_{\mathsf{FRW}} = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right] \tag{26}$$

where the function $a(t)$ is known as the scale factor in 'cosmic' (co-moving) co-ordinates (t, r, θ, ϕ), and k is the FRW topology index, $k = (-1, 0, +1)$. Accordingly, the FRW metric (26) admits a 6-dimensional isometry group G that is either $SO(1,3)$, $E(3)$ or $SO(4)$, acting on the orbits $G/SO(3)$, with the spatial 3-dimensional sections H^3, E^3 or S^3, respectively. Important notice: $C^{\mathsf{FRW}}_{ijkl} = 0$

- Inflation is defined as the early universe epoch during which

$$\ddot{a}\,(t) > 0\,, \text{ or equivalently }, \ \frac{d}{dt}\left(\frac{H^{-1}}{a}\right) < 0 \tag{27}$$

where $H = \dot{a}\,/a$ is Hubble 'constant'. The amount of inflation (# e-foldings) is

$$N = \ln \frac{a(t_{\mathsf{end}})}{a(t_{\mathsf{start}})} = \int_{t_{\mathsf{start}}}^{t_{\mathsf{end}}} H\, dt \tag{28}$$

Modified cosmological (Friedmann) equations

Substituting the FRW metric into a generic (about 100 different terms!) quartic gravity equations of motion yields the generalized Friedmann equation having the form ($k = 0$)

$$3H^2 \equiv 3\left(\frac{\dot{a}}{a}\right)^2 = \beta P_8\left(\frac{\dot{a}}{a}, \frac{\ddot{a}}{a}, \frac{\dddot{a}}{a}, \frac{\ddddot{a}}{a}\right) \quad , \tag{29}$$

where P_8 is a polynomial with respect to its arguments,

$$P_8 = \sum_{\substack{n_1+2n_2+3n_3+4n_4=8, \\ n_1,n_2,n_3,n_4\geq 0}} c_{n_1 n_2 n_3 n_4} \left(\frac{\dot{a}}{a}\right)^{n_1} \left(\frac{\ddot{a}}{a}\right)^{n_2} \left(\frac{\dddot{a}}{a}\right)^{n_3} \left(\frac{\ddddot{a}}{a}\right)^{n_4} \tag{30}$$

The sum goes over the integer partitions $(n_1, 2n_2, 3n_3, 4n_4)$ of 8, the dots stand for the derivatives with respect to time t, and $c_{n_1 n_2 n_3 n_4}$ are some real coefficients. The highest derivative enters linearly at most, $n_4 = 0, 1$.

For instance, the FRW Ansatz with $k = 0$ results in the non-vanishing curvatures

$$R^0{}_{\mu 0\nu} = \delta_{\mu\nu}\, \ddot{a}\, a, \;\; R^\mu{}_{\nu\lambda\rho} = \left(\delta^\mu_\lambda \delta_{\nu\rho} - \delta^\mu_\rho \delta_{\nu\lambda}\right) (\dot{a})^2, \;\; R^\mu_\nu = -\delta^\mu_\nu \left[\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2\right] \tag{31}$$

where $\mu, \nu, \lambda, \rho = 1, 2, 3$. As regards the $(BR)^2$ gravity, we find

$$3H^2 + \beta \left[9\left(\frac{\ddot{a}}{a}\right)^4 - 36H^2\left(\frac{\ddot{a}}{a}\right)^3 + 84H^4\left(\frac{\ddot{a}}{a}\right)^2 - 36H\left(\frac{\ddot{a}}{a}\right)^2\left(\frac{\dddot{a}}{a}\right)\right.$$
$$\left. + 63H^8 - 72H^3\left(\frac{\ddot{a}}{a}\right)\left(\frac{\dddot{a}}{a}\right) + 48H^6\left(\frac{\ddot{a}}{a}\right) - 24H^5\left(\frac{\dddot{a}}{a}\right)\right] = 0 \tag{32}$$

It is remarkable that the 4th order time derivatives cancel, whereas the square of the 3rd order time derivative of the scale factor, $\dddot{a}^{\,2}$, does not appear at all.

Exact non-perturbative inflationary solutions

The structure of eqs. (29) and (30) admits the existence of rather generic exact inflationary solutions without a spacetime singularity, when using the most naive (de Sitter) Ansatz for the scale factor,

$$a(t) = a_0 e^{Bt} \tag{33}$$

with some real positive constants a_0 and B. Substituting eq. (33) into eq. (29), we get $3B^2 = (\#)\beta B^8$, whose coefficient $(\#)$ is just a sum of all c-coefficients in eq. (30). Assuming the $(\#)$ to be positive, we find (33) as the exact solution with

$$B = \left(\frac{3}{\#\beta}\right)^{1/6} \tag{34}$$

This solution in non-perturbative in β, i.e. it is impossible to get it when considering the quartic curvature terms as a perturbation. The assumption that we are dealing with the leading correction, implies $Bt \ll 1$. Because of eqs. (11) and (34), it leads to the natural hierarchy

$$\kappa M_{\mathsf{KK}} \ll 1 \quad \text{or} \quad l_{\mathsf{Pl}} \ll l_{\mathsf{KK}} \tag{35}$$

where we have introduced the four-dimensional Planck scale $l_{\rm Pl} = \kappa$ and the compactification scale $l_{\rm KK} = M_{\rm KK}^{-1}$.

The effective Hubble scale B of eq. (34) should be lower than the effective (with warping) KK scale $M_{\rm KK}^{\rm eff.} = e^A M_{\rm KK}$, in order to validate our four-dimensional description of gravity, i.e. our ignorance of all KK modes,

$$B < M_{\rm KK}^{\rm eff.} \tag{36}$$

It rules out the naive KK reduction (with $A = 0$) but still allows the warped compactification (7), when the average warp factor is tuned as

$$e^A < \frac{(\kappa M_{\rm KK})^{2/5}}{(9/\#)^{3/10} 2^{23/10} \pi} \sim \mathcal{O}\left(10^{-3}\right) \tag{37}$$

where we have used eq. (11) and estimated $(\#)$ as of order 10.

Our exact solution (33) is non-singular, while it describes an inflationary isotropic and homogeneous universe; cf. Starobinsky (1980), Maeda, Ohta (2004)

Stability analysis

To be truly inflationary solutions, eqs. (33) and (34) should correspond to the stable fixed points (or attractors). When using the parametrization

$$a(t) = e^{\lambda(t)} \tag{38}$$

we find

$$\begin{aligned}
\frac{\dot{a}}{a} &= \dot{\lambda} \ , \\
\frac{\ddot{a}}{a} &= \ddot{\lambda} + (\dot{\lambda})^2 \ , \\
\frac{\dddot{a}}{a} &= \dddot{\lambda} + 3\,\ddot{\lambda}\dot{\lambda} + (\dot{\lambda})^3 \ , \\
\frac{\ddddot{a}}{a} &= \ddddot{\lambda} + 4\,\dddot{\lambda}\dot{\lambda} + 6\,\ddot{\lambda}\,(\dot{\lambda})^2 + 3(\ddot{\lambda})^2 + (\dot{\lambda})^4
\end{aligned} \tag{39}$$

The polynomial (30) now takes the form

$$P_8 = \sum_{\substack{n_1+2n_2+3n_3+4n_4=8, \\ n_1,n_2,n_3,n_4\geq 0}} d_{n_1 n_2 n_3 n_4} \left(\dot{\lambda}\right)^{n_1} \left(\ddot{\lambda}\right)^{n_2} \left(\dddot{\lambda}\right)^{n_3} \left(\ddddot{\lambda}\right)^{n_4} \tag{40}$$

where the d-coefficients are some simple linear combinations of the c-coefficients.

Equations (33) and (34) are also simplified as

$$\lambda(t) = Bt + \lambda_0 \ , \qquad \text{where} \qquad a_0 = e^{\lambda_0} \quad \text{and} \quad d_{8000} = \# \tag{41}$$

The equations of motion (29) can be rewritten to the form

$$3y_1^2 = \beta P_8(y_1, y_2, y_3, \dot{y}_3) \equiv \beta P_{8,0}(y_1, y_2, y_3) + \beta P_4(y_1, y_2, y_3)\, \dot{y}_3 \ , \tag{42}$$

where we have introduced the notation

$$y_1 = \dot{\lambda} \ , \quad y_2 = \ddot{\lambda} \ , \quad y_3 = \dddot{\lambda} \ . \tag{43}$$

Equation (42) can now be put into the autonomous form

$$\begin{aligned} \dot{y}_1 &= y_2 \ , \\ \dot{y}_2 &= y_3 \ , \\ \dot{y}_3 &= \frac{3y_1^2 - \beta P_{8,0}(y_1, y_2, y_3)}{\beta P_4(y_1, y_2, y_3)} \equiv f(y_1, y_2, y_3) \end{aligned} \tag{44}$$

that is quite suitable for the stability analysis against small perturbations about the fixed point, $y_a = y_a^{\text{fixed}} + \delta y_a$, where $a = 1, 2, 3$.
We find

$$\begin{aligned}
\delta \dot{y}_1 &= \delta y_2 \ , \\
\delta \dot{y}_2 &= \delta y_3 \ , \\
\delta \dot{y}_3 &= \frac{\partial f}{\partial y_1}\Big| \delta y_1 + \frac{\partial f}{\partial y_2}\Big| \delta y_2 + \frac{\partial f}{\partial y_3}\Big| \delta y_3 \ ,
\end{aligned} \tag{45}$$

where all the partial derivatives are taken at the fixed point (denoted by $|$). The fixed point is stable when all the eigenvalues of the matrix

$$\widehat{M} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{\partial f}{\partial y_1}\Big| & \frac{\partial f}{\partial y_2}\Big| & \frac{\partial f}{\partial y_3}\Big| \end{pmatrix} \tag{46}$$

in eq. (45) are negative or have negative real parts. Then our fixed point is a stable attractor.

Scale factor duality

More restrictions on the cosmological polynomial P_8 of eq. (30) arise in specific superstring-inspired cosmological scenarios, e.g. in the pre-big bang scenario of Gasperini-Veneziano. They imposed the scale factor duality, by requiring the cosmological Friedmann equation to be invariant under the duality transformation

$$a(t) \leftrightarrow \frac{1}{a(t)} \equiv b(t) \tag{47}$$

This duality is a cosmological version of the genuine stringy T-duality (which is a symmetry of the non-perturbative string spectrum), in the case of time-dependent backgrounds. The scale factor duality is merely the symmetry of the perturbative equations of motion of the background fields. It is used to exclude the cosmological singularity (big bang) in the pre-big bang scenario.

In the λ-parametrization (38) the scale-factor duality transformation (47) takes the very simple form

$$\lambda(t) \leftrightarrow -\lambda(t) \tag{48}$$

The equations of motion in the form (42) are manifestly invariant under the transform $\lambda(t) \to \lambda(t) + \lambda_0$, where λ_0 is an arbitrary constant. We also find

$$\begin{aligned}
\frac{\dot{a}}{a} &= -\frac{\dot{b}}{b}\,, \quad \frac{\ddot{a}}{a} = -\frac{\ddot{b}}{b} + 2\left(\frac{\dot{b}}{b}\right)^2\,, \\
\frac{\dddot{a}}{a} &= -\frac{\dddot{b}}{b} + 6\left(\frac{\dot{b}}{b}\right)\left(\frac{\ddot{b}}{b}\right) - 6\left(\frac{\dot{b}}{b}\right)^3\,, \\
\frac{\ddddot{a}}{a} &= -\frac{\ddddot{b}}{b} + 6\left(\frac{\ddot{b}}{b}\right)^2 + 8\left(\frac{\dot{b}}{b}\right)\left(\frac{\dddot{b}}{b}\right) - 36\left(\frac{\dot{b}}{b}\right)^2\left(\frac{\ddot{b}}{b}\right) + 24\left(\frac{\dot{b}}{b}\right)^4
\end{aligned} \tag{49}$$

To illustrate, how the scale factor duality affects the polynomial P_8, let's consider the case with the 3rd order time derivatives, motivated by eq. (32), in the notation

$$\frac{\dot{a}}{a} = x\,, \quad \frac{\ddot{a}}{a} = y\,, \quad \frac{\dddot{a}}{a} = z \tag{50}$$

The duality invariance condition reads

$$P_8(-x, 2x^2 - y, 6xy - 6x^3 - z) = P_8(x, y, z) \tag{51}$$

The structure of the polynomial P_8 in eq. (30), as the sum over partitions of 8, restricts a solution to eq. (51) to be most quadratic in z,

$$P_8(x, y, z) = a_2(x, y)z^2 + b_5(x, y)z + c_8(x, y) \ , \tag{52}$$

whose coefficients are polynomials in (x, y), of order given by subscript,

$$\begin{aligned} a_2(x, y) &= a_0x^2 + a_1y \ , \\ b_5(x, y) &= b_0x^5 + b_1x^3y + b_2xy^2 \ , \\ c_8(x, y) &= c_4y^4 + c_3y^3x^2 + c_2y^2x^4 + c_1yx^6 + c_0x^8 \end{aligned} \tag{53}$$

Their substitution into eq. (51) yields an overdetermined system of linear equations on the coefficients. Nevertheless, we find a consistent general solution,

$$\begin{aligned} P_8(x, y, z) = a_0x^2z^2 &+ (b_0x^5 - 3a_0xy^2)z \\ &+ c_4y^4 + (9a_0 - 4c_4)y^3x^2 + c_2y^23x^4 \\ &+ (8c_4 - 18a_0 - 3b_0 - 2c_2)yx^6 + c_0x^8 \end{aligned} \tag{54}$$

parameterized by five real coefficients $(a_0, b_0, c_4, c_2, c_0)$.

Demanding the existence of the exact solution (33), i.e. positivity of $(\#)$ in eq. (34), yields

$$5c_4 + c_0 > 11a_0 + 2b_0 + c_2 \tag{55}$$

- As regards the $(BR)^2$ gravity representing the 'minimal' candidate for the off-shell superstring quartic effective action, we checked that neither the duality invariance nor the inequality (55) are satisfied by the coefficients present in eq. (32). We interpret it as the clear indications that some additional Ricci-dependent terms have to be added to the $(BR)^2$ terms or, equivalently, the $(BR)^2$ gravity is ruled out as an off-shell quartic effective action for superstrings.

- Our results are generalizable to any higher order in the spacetime curvatures, because it amounts to increasing the order of the polynomial P. It leads to the ALL-orders or non-perturbative speculative cosmological equation:

$$H^2 = \frac{\dot{a}^2}{a^2} = \beta P[a(t)] \tag{56}$$

whose function P is subject to the scale-factor duality condition

$$P[a(t)] = P[1/a(t)] \ . \tag{57}$$

From the quartic supergravity to a non-polynomial supergravity

Basic facts about 4-dim, $N = 1$ supergravity in superspace

A concise and manifestly supersymmetric description of supergravity is given by Superspace. We use here the natural units $c = \hbar = \kappa = 1$.

The chiral superspace density (in the supersymmetric gauge-fixed form) reads

$$\mathcal{E}(x,\theta) = e(x)\left[1 - 2i\theta\sigma_a\bar{\psi}^a(x) + \theta^2 B(x)\right] , \tag{58}$$

where $e = \sqrt{-\det g_{\mu\nu}}$, $g_{\mu\nu}$ is a spacetime metric, $\psi^a_\alpha = e^a_\mu \psi^\mu_\alpha$ is a chiral gravitino, $B = S - iP$ is the complex scalar auxiliary field. We use the lower case middle greek letters $\mu, \nu, \ldots = 0, 1, 2, 3$ for curved spacetime vector indices, the lower case early latin letters $a, b, \ldots = 0, 1, 2, 3$ for flat (target) space vector indices, and the lower case early greek letters $\alpha, \beta, \ldots = 1, 2$ for chiral spinor indices.

The solution of the superspace Bianchi identitiies and the constraints defining the $N = 1$ Poincaré-type *minimal* supergravity results in only three relevant superfields $\mathcal{R}$, $\mathcal{G}_a$ and $\mathcal{W}_{\alpha\beta\gamma}$, subject to the off-shell relations:

$$\mathcal{G}_a = \bar{\mathcal{G}}_a \ , \qquad \mathcal{W}_{\alpha\beta\gamma} = \mathcal{W}_{(\alpha\beta\gamma)} \ , \qquad \bar{\nabla}_{\dot{\alpha}}\mathcal{R} = \bar{\nabla}_{\dot{\alpha}}\mathcal{W}_{\alpha\beta\gamma} = 0 \ , \tag{59}$$

and

$$\bar{\nabla}^{\dot{\alpha}}\mathcal{G}_{\alpha\dot{\alpha}} = \nabla_\alpha\mathcal{R} \ , \qquad \nabla^\gamma\mathcal{W}_{\alpha\beta\gamma} = \tfrac{i}{2}\nabla_\alpha{}^{\dot{\alpha}}\mathcal{G}_{\beta\dot{\alpha}} + \tfrac{i}{2}\nabla_\beta{}^{\dot{\alpha}}\mathcal{G}_{\alpha\dot{\alpha}} \ , \tag{60}$$

where $(\nabla_\alpha, \bar{\nabla}_{\dot{\alpha}}.\nabla_{\alpha\dot{\alpha}})$ represent the curved superspace $N = 1$ supercovariant derivatives, and bars denote complex conjugation.

The covariantly chiral complex scalar superfield $\mathcal{R}$ has the scalar curvature R as the coefficient at its θ^2 term, the real vector superfield $\mathcal{G}_{\alpha\dot{\alpha}}$ has the traceless Ricci tensor, $R_{\mu\nu}+R_{\nu\mu}-\frac{1}{2}g_{\mu\nu}R$, as the coefficient at its $\theta\sigma^a\bar{\theta}$ term, whereas the covariantly chiral, complex, totally symmetric, fermionic superfield $\mathcal{W}_{\alpha\beta\gamma}$ has the Weyl tensor $W_{\alpha\beta\gamma\delta}$ as the coefficient at its linear θ^δ-dependent term. A generic higher-derivative supergravity Lagrangian (e.g., representing the supergravitational part of the superstring effective action) is given by

$$\mathcal{L} = \mathcal{L}(\mathcal{R}, \mathcal{G}, \mathcal{W}, \ldots) \tag{61}$$

where the dots stand for arbitrary supercovariant derivatives of the supergravity superfields in superspace.

New proposal: $F(\mathcal{R})$ supergravity

Let's concentrate on the particular sector of the generic higher-derivative supergravity (61), by ignoring the tensor superfields $\mathcal{W}_{\alpha\beta\gamma}$ and $\mathcal{G}_{\alpha\dot{\alpha}}$, as well as the derivatives of the scalar superfield $\mathcal{R}$:

$$S_F = \int d^4x d^2\theta \, \mathcal{E} F(\mathcal{R}) + \text{H.c.} \tag{62}$$

with a holomorphic function $F(\mathcal{R})$. Besides manifest local $N = 1$ supersymmetry, the action (62) also possess the auxiliary freedom, since the auxiliary field B does not propagate. It distinguishes the action (62) from other possible truncations of eq. (61). The action (62) gives rise to the spacetime torsion generated by gravitino, while its bosonic terms have the form

$$S_f = \int d^4x \sqrt{-g} \, f(R) \ . \tag{63}$$

Hence, eq. (62) can also be considered as a locally $N = 1$ supersymmetric extension of the $f(R)$-type modified gravity, widely used to 'explain' the present Universe acceleration.

Getting fields from superfields

Appplying the superspace chiral density formula

$$\int d^4x d^2\theta\, \mathcal{E}\mathcal{L} = \int d^4x\, e\, \{\mathcal{L}_{\text{last}} + B\mathcal{L}_{\text{first}}\} \tag{64}$$

to our action (62) yields its bosonic part as

$$F'(\bar{X})\left[\tfrac{1}{3}R_* + 4\bar{X}X\right] + 3XF(\bar{X}) + \text{H.c.} \tag{65}$$

where primes denote differentiation. We have used the notation

$$X = \tfrac{1}{3}B\,, \qquad R_* = R - iR_{\text{tor}} \tag{66}$$

Varying eq. (65) with respect to the auxiliary fields X and $\bar{X}$ gives rise to an algebraic equation on the auxiliary fields,

$$3\bar{F} + X(4\bar{F}' + 7F') + 4\bar{X}XF'' + \tfrac{1}{3}F''R_* = 0 \tag{67}$$

- Let's consider the simple special case first, when

$$F'' = 0 \qquad \text{or, equivalently,} \qquad F(\mathcal{R}) = f_0 + f_1\mathcal{R} \tag{68}$$

with some complex constants f_0 and f_1, where $\mathrm{Re} f_1 < 0$. Then eq. (67) is easily solved as

$$\bar{X} = \frac{-3(f_0 + f_1 R_*)}{4 f_1 + 7 \bar{f}_1} \tag{69}$$

Substituting the solution (69) back into the Lagrangian (65) yields

$$\tfrac{2}{3}(\mathrm{Re} f_1) R_* - \frac{9\,|f_0|^2}{14(\mathrm{Re} f_1)} \equiv -\frac{1}{2\kappa^2} R_* - \Lambda = -\frac{1}{2\kappa^2} R(\Gamma + T) - \Lambda \tag{70}$$

where we have introduced the standard gravitational constant $\kappa_0 = M_{\mathrm{Planck}}^{-1}$ in terms of the (reduced) Planck mass, the standard supergravity connection (i.e. Christoffel symbols Γ plus torsion T), and a cosmological constant Λ,

$$\kappa = \sqrt{\frac{3}{4\,|\mathrm{Re} f_1|}} \quad , \qquad \Lambda = \frac{-9\,|f_0|^2}{14\,|\mathrm{Re} f_1|} \tag{71}$$

As is clear from the above equations, the cosmological constant in the standard supergravity is always negative. It is all different in the modified supergravity with $F'' \neq 0$.

Supersymmetric Legendre-Weyl-Kähler transformation

The superfield action (62) is classically equivalent to

$$S_V = \int d^4x d^2\theta\, \mathcal{E}\, [\mathcal{Z}\mathcal{R} - V(\mathcal{Z})] + \text{H.c.} \tag{72}$$

with the covariantly chiral superfield $\mathcal{Z}$ as a Lagrange multiplier. Varying the action (72) with respect to $\mathcal{Z}$ gives back the original action (62) provided that

$$F(\mathcal{R}) = \mathcal{R}\mathcal{Z}(\mathcal{R}) - V(\mathcal{Z}(\mathcal{R})) \tag{73}$$

where the function $\mathcal{Z}(\mathcal{R})$ is defined by inverting the function

$$\mathcal{R} = V'(\mathcal{Z}) \tag{74}$$

Equations (73) and (74) define the superfield Legendre transform, and imply

$$F'(\mathcal{R}) = Z(\mathcal{R}) \qquad \text{and} \qquad F''(\mathcal{R}) = Z'(\mathcal{R}) = \frac{1}{V''(\mathcal{Z}(\mathcal{R}))} \tag{75}$$

where $V'' = d^2V/d\mathcal{Z}^2$. The second formula (75) is the duality relation between the supergravitational function F and the chiral superpotential V.

A super-Weyl transform of the acton (72) can be done entirely in superspace. In terms of components, the super-Weyl transform amounts to a Weyl transform, a chiral rotation and a (superconformal) S-supersymmetry transformation (Howe). The chiral density superfield $\mathcal{E}$ is a chiral compensator of the super-Weyl transformations,

$$\mathcal{E} \to e^{3\Phi}\mathcal{E} \,, \tag{76}$$

whose parameter Φ is an arbitrary covariantly chiral superfield, $\bar{\nabla}_{\dot{\alpha}}\Phi = 0$. Under the transformation (76) the covariantly chiral superfield $\mathcal{R}$ transforms as

$$\mathcal{R} \to e^{-2\Phi}\left(\mathcal{R} - \tfrac{1}{4}\bar{\nabla}^2\right)e^{\bar{\Phi}} \quad . \tag{77}$$

The super-Weyl chiral superfield parameter Φ can be traded for the chiral Lagrange multiplier $\mathcal{Z}$ by using a generic gauge condition

$$\mathcal{Z} = \mathcal{Z}(\Phi) \tag{78}$$

where $\mathcal{Z}(\Phi)$ is a holomorphic function of Φ. It results in the equivalent action

$$S_\Phi = \int d^4x d^4\theta\, E^{-1} e^{\Phi+\bar{\Phi}}\left[\mathcal{Z}(\Phi) + \text{H.c.}\right] - \int d^4x d^2\theta\, \mathcal{E} e^{3\Phi} V(\mathcal{Z}(\Phi)) + \text{H.c.} \tag{79}$$

Equation (79) has the standard form of the action of a chiral matter superfield coupled to supergravity,

$$S[\Phi,\bar{\Phi}] = \int d^4x d^4\theta\, E^{-1}\Omega(\Phi,\bar{\Phi}) + \left[\int d^4x d^2\theta\, \mathcal{E} P(\Phi) + \text{H.c.}\right] , \qquad (80)$$

in terms of a 'Kähler' potential $\Omega(\Phi,\bar{\Phi})$ and a chiral superpotential $P(\Phi)$. In our case (79) we find

$$\Omega(\Phi,\bar{\Phi}) = e^{\Phi+\bar{\Phi}}\left[\mathcal{Z}(\Phi)+\bar{\mathcal{Z}}(\bar{\Phi})\right] , \quad P(\Phi) = -e^{3\Phi}V(\mathcal{Z}(\Phi)) \ . \qquad (81)$$

The truly Kähler potential $K(\Phi,\bar{\Phi})$ is given by

$$K = -3\,\text{ln}(-\frac{\Omega}{3}) \quad \text{or} \quad \Omega = -3e^{-K/3} , \qquad (82)$$

because of the invariance of the action (80) under the supersymmetric Kähler-Weyl transformations

$$K(\Phi,\bar{\Phi}) \to K(\Phi,\bar{\Phi}) + \Lambda(\Phi) + \bar{\Lambda}(\bar{\Phi}) , \quad \mathcal{E} \to e^{\Lambda(\Phi)}\mathcal{E} , \qquad (83)$$

$P(\Phi) \to -e^{-\Lambda(\Phi)}P(\Phi)$, with an arbitrary chiral superfield parameter $\Lambda(\Phi)$.

Scalar potential

(in components) is given by the standard formula (Cremmer *et al*)

$$\mathcal{V}(\phi,\bar{\phi}) = e^{\Omega}\left\{\left|\frac{\partial P}{\partial \Phi} + \frac{\partial \Omega}{\partial \Phi}P\right|^2 - 3\,|P|^2\right\}\Big| \ , \tag{84}$$

where all superfields are restricted to their leading field components, $\Phi| = \phi(x)$. Equation (84) can be simplified by making use of the Kähler-Weyl invariance (83) that allows us to choose the gauge

$$P = 1 \tag{85}$$

It is equivalent to the well known fact that the scalar potential (84) is actually governed by the single (Kähler-Weyl-invariant) potential

$$G(\Phi,\bar{\Phi}) = \Omega + \ln P + \ln \bar{P} \tag{86}$$

In our case (81) we have

$$G = e^{\Phi+\bar{\Phi}}\left[\mathcal{Z}(\Phi) + \bar{\mathcal{Z}}(\bar{\Phi})\right] + 3(\Phi+\bar{\Phi}) + \ln(-V(\mathcal{Z}(\Phi)) + \ln(-\bar{V}(\bar{\mathcal{Z}}(\bar{\Phi})) \tag{87}$$

Let's now specify our gauge (78) by choosing the condition

$$3\Phi + \ln(-V(\mathcal{Z}(\Phi)) = 0 \quad \text{or} \quad V(\mathcal{Z}(\Phi)) = -e^{-3\Phi} \tag{88}$$

that is equivalent to eq. (85). Then the potential (87) gets simplified to

$$G = \Omega = e^{\Phi+\bar{\Phi}}\,[\mathcal{Z}(\Phi) + \bar{\mathcal{Z}}(\bar{\Phi})] \tag{89}$$

Equations (73), (74) and (89) are the one-to-one algebraic relations between a holomorphic function $F(\mathcal{R})$ in our modified supergravity action (62) and a holomorphic function $\mathcal{Z}(\Phi)$ defining the scalar potential (84)

$$\mathcal{V} = e^{G}\left[\left(\frac{\partial^2 G}{\partial\Phi\partial\bar{\Phi}}\right)^{-1}\frac{\partial G}{\partial\Phi}\frac{\partial G}{\partial\bar{\Phi}} - 3\right]\Bigg| \tag{90}$$

in the classically equivalent scalar-tensor supergravity. The latter can used for embedding the standard slow-roll inflation into supergravity. In our setup that correspondence can be promoted further, by embedding the slow-roll inflation into the 'purely geometrical' (apparently) higher-derivative supergravity theory (62), defined in terms of a single holomorphic function.

No-scale modified supergravity

The no-scale supergravity arises by demanding the scalar potential (90) to vanish. It results in the vanishing cosmological constant without fine-tuning. The no-scale supergravity potential G has to obey the non-linear 2nd-order partial differential equation

$$3\frac{\partial^2 G}{\partial\Phi\partial\bar{\Phi}} = \frac{\partial G}{\partial\Phi}\frac{\partial G}{\partial\bar{\Phi}} \tag{91}$$

A gravitino mass $m_{3/2}$ is given by the vacuum expectation value (Wess, Bagger)

$$m_{3/2} = \left\langle e^{G/2} \right\rangle \tag{92}$$

so that the spontaneous supersymmetry breaking scale can be chosen at will.

A widely used exact solution to eq. (91) is given by

$$G = -3\ln(\Phi + \bar{\Phi}) \ . \tag{93}$$

In more recent literature, the no-scale solution (93) is usually modified by other terms, in order to achieve a universe with a positive cosmological constant (uplifting) — as e.g., in the KKLT mechanism (Kachru *et al*).

To appreciate the difference between the standard no-scale supergravity solution and our modified supergravity, let's notice that the *Ansatz* (93) is inconsistent with our potential (89) by any choice of the function $\mathcal{Z}$. Instead, in our case, demanding eq. (91) gives rise to the 1st-order non-linear differential equation

$$3\left(e^{\bar{\Phi}}X' + e^{\Phi}\bar{X}'\right) = \left|e^{\bar{\Phi}}X' + e^{\Phi}\bar{X}\right|^2 \tag{94}$$

where we have introduced the notation

$$\mathcal{Z}(\Phi) = e^{-\Phi}X(\Phi)\,, \qquad X' = \frac{dX}{d\Phi} \tag{95}$$

Accordingly, the gravitino mass (92) is given by

$$m_{3/2} = \left\langle \exp\frac{1}{2}\left(e^{\bar{\Phi}}X + e^{\Phi}\bar{X}\right)\right\rangle \tag{96}$$

- We are not aware of any non-trivial holomorphic exact solution to eq. (94).

For instance, should it obey the holomorphic differential equation

$$X' = e^{\Phi}g(X,\Phi) \tag{97}$$

with a holomorphic function $g(X,\Phi)$, eq. (94) gives rise to a functional equation,

$$3\,(g+\bar{g}) = \left|e^{\bar{\Phi}}g + \bar{X}\right|^2 \tag{98}$$

- Being restricted to the real variables $\Phi = \bar{\Phi} \equiv y$ and $X = \bar{X} \equiv x$, eq. (94) reads

$$6x' = e^y(x'+x)^2\,, \quad \text{where} \quad x' = \frac{dx}{dy} \tag{99}$$

This equation can be integrated after a change of variables *

$$x = e^{-y}u \tag{100}$$

that leads to a quadratic equation with respect to $u' = du/dy$,

$$(u')^2 - 6u' + 6u = 0 \tag{101}$$

It follows

$$y = \int^u \frac{d\xi}{3 \pm \sqrt{3(3-2\xi)}} = \mp\sqrt{1-\tfrac{2}{3}u} + \ln\left(\sqrt{3(3-2u)} \pm 3\right) + C \tag{102}$$

*I am grateful to A. Starobinsky who pointed it out to me

Conclusion

- A higher-derivative gravity defies the Hawking-Penrose theorem about the existence of a spacetime singularity in any exact solution to Einstein equations. The initial cosmological singularity can be avoided by including the higher order curvature terms on equal footing with the Einstein-Hilbert term

- Our results predict possible existence of a very short de Sitter phase driven by the quartic curvature terms, provided that the gravitational terms dominated over all matter contributions at the beginning of inflation in a very early universe

- We proposed a locally $N = 1$ supersymmetric extension of modified $f(R)$ gravity, in terms of a single holomorphic function, and proved its classical equivalence to the standard theory of a chiral scalar superfield (with a non-trivial chiral superpotential), coupled to the (minimal) Poincaré supergravity in four space-time dimensions. This statement can be used as the technical framework for various cosmological scenarios unifying the early universe inflation and the present universe acceleration

- We speculated on a possible origin of our modified supergravity as the effective supergravity generated by quantum superstrings (or M-theory), with a dilaton-axion field as the leading field component of the chiral superfield in the dual picture

- We computed a scalar potential in the dual version of our modified supergravity via the Legendre–Kähler-Weyl transform in superspace. The Kähler potential, the superpotential and the scalar potential of the dual theory are all governed by a single holomorphic function

- We also found the conditions for the vanishing cosmological constant and spontanenous supersymmetry breaking, without fine-tuning, defining the no-scale $F(\mathcal{R})$ modified supergravity.

Discussion and Outlook

- Finding a Graceful Exit from the geometrical inflation to a matter-driven inflation, as well as getting the right number of e-foldings, need further investigation

- A possibility to get inflation by modifying Einstein equations with the 2nd-order curvature terms (representing the gravitational anomalies of matter fields) was first discovered a long time ago by Starobinsky (1980). A similar mechanism is known in the four-dimensional supergravity, with inflation being generated by the $\mathcal{R}^2$-term originating from the one-loop Kähler anomaly (Cardoso, Ovrut)
- Instabilities in the cosmological scenarios based on the 2nd-order curvature terms, against adding higher order scalar curvature terms, were analyzed by Maeda

- In General Relativity, only the spin-2 part of a metric is dynamical. A dynamical generation of a massive scalar field is known to occur already in the presence of the quadratic curvature terms (Buchbinder *et al*) out of the spin-0 part of the metric. In supergravity, as we found, the whole chiral scalar superfield becomes dynamical, while it can be identified with the super-Weyl compensator

- In superstring theory, the superspin-0 part of the supervielbein is given by a chiral scalar superfield, whose leading complex component represents a *dilaton-axion* field, $\phi| = \varphi(x) + iB(x)$. It is tempting to identify $\varphi(x)$ with a superstring dilaton, and B with a superstring B-field (or axion). In string theory, the dilaton field controls the superstring loops and (D-brane) instantons, which may be the source of the function $F(\mathcal{R})$. The B-field is the source of the non-minimal space-time torsion in string theory

- Unfortunately, the string theory technology at present does not allow us to compute the function $F(\mathcal{R})$ in eq. (62). It is mainly because of the on-shell nature of the known string theory. However, its $\mathcal{R}$-dependence may be fixed by some additional (off-shell) physical requirements such as no-ghosts, stability, and the scale-factor self-duality, or by going to the IR-limit (weak gravity). For instance, by using the hameleon effect, it was recently demonstrated by Starobinsky that the function

$$f(R) = R + \lambda R_0 \left[\frac{1}{\left(1 + \frac{R^2}{R_0^2}\right)^n} - 1 \right] \qquad (103)$$

with some parameters $R_0 \sim H_0^2$, $\lambda > 0$ and $n > 0$, is fully consistent with all Solar System observations, being also capable of 'explaining' the present Universe acceleration

- There are many other possible choices, e.g., by a *reconstruction* of functiona $f(R)$ or $F(\mathcal{R})$ from a desired scale factor (history) $a(t)$, via the gravitational equations of motion with

$$R = -6\left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right] \tag{104}$$

The superfield extension of eq. (104) at $k = 0$ is given by a superconformally-flat superspace with

$$\mathcal{R} = -\frac{1}{4}e^{-2\kappa\Phi}\bar{\nabla}^2 e^{\kappa\bar{\Phi}} \tag{105}$$

- A generic modified supergravity potential (89) in terms of the holomorphic pre-potential $X(\Phi)$ defined by eq. (95),

$$G = \Omega = e^{\bar{\Phi}}X(\Phi) + e^{\Phi}\bar{X}(\bar{\Phi}) \tag{106}$$

has essentially the same structure as the supergravity potential of $N = 2$ extended supergravity (coupled to $N = 2$ vector matter) that is also governed by a holomorphic pre-potential. We do not know whether it is merely a coincidence or not!

- Sometimes, a positive cosmological constant and a slow-roll inflation are achieved by demanding a shift symmetry of the Kähler potential (i.e. its flatness in one direction), and then perturbing the scalar potential around that flat direction (work in progress)

- In the context of superstrings, the effective supergravity potential (89) may capture some 'stringy' features, such as T-duality and maximal curvature (Gates, Ketov)

- Since we used the (old) *minimal* formulation of an off-shell supergravity multiplet, with a fixed superconnection, adding the minimal coupling to a (scalar- or vector-type) supermatter is straightforward in superspace, just by using the supercovariant derivatives, while it does not change our results. However, adding a non-minimal coupling of matter to the scalar supercurvature would drastically change everything, so it deserves a separate investigation (work in progress).