Enhancing non-Gaussianities by breaking local Lorentz invariance

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The universe today is filled with a bewildering variety of stuff

But in its infancy, it seems to have been far simpler, differing very little from one place to another

Yet even at the earliest times that we can observe directly, there has always been *some* spatial variation

How did these *primordial fluctuations* first arise?

Inflation & primordial perturbations

Inflation is a mechanism for generating variations in the space-time background itself

It relies on just two ingredients:

1) a quantum field, $\phi(t,\mathrm{x})$

2) an expanding universe,

$$ds^2 = dt^2 - a^2(t)\, d\mathrm{x}\cdot d\mathrm{x}$$

A quantum field *always* fluctuates, so the universe must *inevitably* have spatial variations

$$\langle 0(t)\,|\,\phi(t,\mathrm{x})\phi(t,\mathrm{y})\,|\,0(t)\rangle \neq 0$$

the expansion makes tiny quantum stuff big.

Testing our assumptions

~~Inflation & primordial perturbations~~

Inflation is a mechanism for generating variations in the space-time background itself

It relies on just two ingredients:

Who ordered this?

Quantum Gravity?

1) a quantum field, $\phi(t,\mathrm{x})$

2) an expanding universe,

$$ds^2 = dt^2 - a^2(t)\, d\mathrm{x}\cdot d\mathrm{x}$$

How did the universe find itself in this metric?

A quantum field *always* fluctuates, so the universe must *inevitably* have spatial variation,

Which quantum state?

$$\langle 0(t)\,|\,\phi(t,\mathrm{x})\phi(t,\mathbf{y})\,|\,0(t)\rangle \neq 0$$

the expansion makes tiny quantum stuff big.

It is possible to have too much of a good thing?

Particle physics

1) we know the background (flat space)

2) we know all the relevant ingredients (up to some energy scale)

3) we have a well defined and well tested theoretical framework (QFT in flat space)

4) gravity is completely negligible

5) experimentally, we can fiddle, smash, create, and measure anything we want (again, up to some energy scale)

Contrasting Particle physics w/early-universe cosmology

1) we probably know the background (~~flat space~~ FRW space-time)

2) we ~~know all~~ can only guess the relevant ingredients & we do not know the scale
(up to some energy scale)

3) we have a ~~well~~ defined and ~~well~~ un-tested QFT in curved space or quantum gravity
theoretical framework (~~QFT in flat space~~)

4) gravity is completely ~~negligible~~ essential

experimentally: look, but do not touch!

5) ~~experimentally, we can fiddle, smash, create, and measure anything we want (again, up to some energy scale)~~

Describing patterns

How do we describe the information in the space-time fluctuations, $\zeta(t,\mathrm{x})$?

We use a set of *classical*, *n*-point functions,

$$\langle 0(t)\,|\,\zeta(t,\mathrm{x}_1)\,\zeta(t,\mathrm{x}_2)\,\cdots\,\zeta(t,\mathrm{x}_n)\,|\,0(t)\rangle$$

These tell how a fluctuation in one place is correlated with those in (several) other places

The fluctuations are *small*, so it is harder and harder to measure the higher-point functions,

$\langle 0(t)\,|\,\zeta(t,\mathrm{x})\,\zeta(t,\mathrm{y})\,|\,0(t)\rangle \;\rightarrow$ power spectrum (measured)

$\langle 0(t)\,|\,\zeta(t,\mathrm{x})\,\zeta(t,\mathrm{y})\,\zeta(t,\mathrm{z})\,|\,0(t)\rangle \;\rightarrow$ "non-Gaussianities"

Generating patterns

Inflation provides the "initial conditions" for a more conventional universe

space-time fluctuations

$$\langle 0(t) \,|\, \zeta(t,\mathrm{x})\zeta(t,\mathrm{y}) \,|\, 0(t) \rangle$$

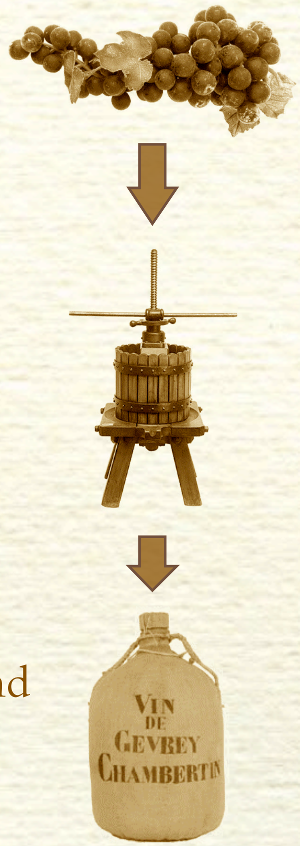
Einstein's Equations
&
Bolzmann's Equations

Large-Scale Structure
(density fluctuations)

$$\langle \delta\rho(t_{\mathrm{x}},\mathrm{x})\delta\rho(t_{\mathrm{y}},\mathrm{y}) \rangle$$

Cosmic Microwave Background
(temperature fluctuations)

$$\langle \delta T(\theta_1,\phi_1)\delta T(\theta_2,\phi_2) \rangle$$



Generating and comparing patterns

Inflation provides the "initial conditions" for a more conventional universe

space-time fluctuations

$\langle 0(t) \,|\, \zeta(t,\mathrm{x})\zeta(t,\mathrm{y}) \,|\, 0(t) \rangle$

try to infer this

Einstein's Equations
&
Bolzmann's Equations

from this

Large-Scale Structure
(density fluctuations)

$\langle \delta\rho(t_{\mathrm{x}},\mathrm{x})\,\delta\rho(t_{\mathrm{y}},\mathrm{y}) \rangle$

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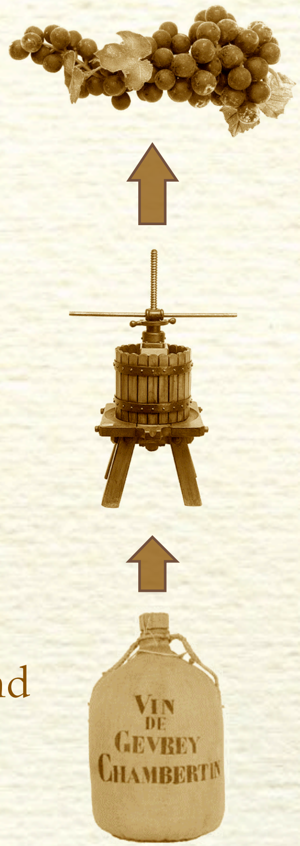

Inflation & primordial perturbations

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Quantum Gravity?

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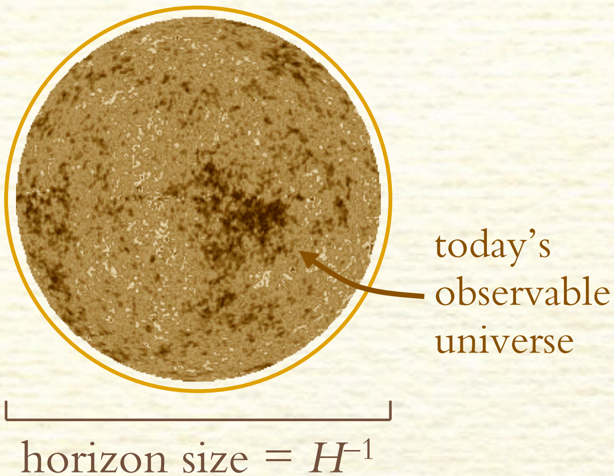
It is possible to have too much of a good thing?

Too much of a good thing

[the trans-Planckian problem of inflation]

To explain our universe today, a fluctuation the size of the observable universe must once have been smaller than the causal horizon during inflation



H = expansion rate during inflation
($< 10^{14}$ GeV)

This *minimally* requires about 60 "*e*-folds" of inflation

Too much of a good thing

[the trans-Planckian problem of inflation]

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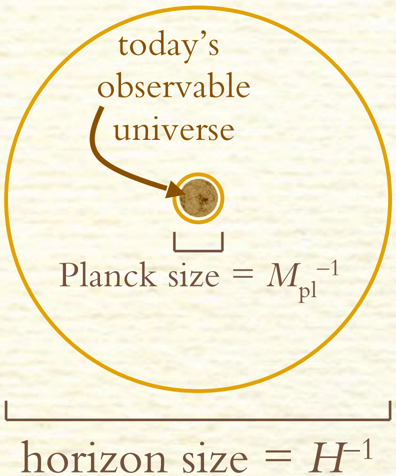


H = expansion rate during inflation ($< 10^{14}$ GeV)

M_{pl} = Planck mass ($\approx 10^{18}$ GeV)

$\ln(M_{pl}/H)$ more e-folds

But with only a bit more, everything would once have been smaller than a *Planck length*

Why is this bad?

Inflation usually assumes an action such as

$$S = \int d^4x \sqrt{-g} \; \left(\tfrac{1}{2} M_{\rm pl}{}^2 R + \tfrac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right)$$

But we are looking at fluctuations, *derivatives*, and higher terms might be important

The standard treatment also assumes a *local*, *point-like*, *locally Lorentz-invariant* structure of *space-time* down to *arbitrarily* small scales

And $\zeta(t,\mathrm{x})$ is not a quantum field in a classical background, it is (in part) the quantum fluctuation of the space-time itself

A case: breaking Lorentz invariance

[Collins & Holman, hep-ph/0905.4925]

Can we use observations to constrain some of these possibilities?

Consider a model that breaks local Lorentz invariance at some scale M, $[H < M < M_{\rm pl}]$

$$\frac{d_1}{M} H^2 \zeta^3 \, , \, \frac{d_2}{a^2 M} \zeta^2 \, \vec{\nabla} \cdot \vec{\nabla} \zeta$$

These operators will contribute to the three-point function,

$$\langle 0(t) \, | \, \zeta(t, \mathrm{x}) \zeta(t, \mathrm{y}) \zeta(t, \mathrm{z}) \, | \, 0(t) \rangle$$

To be visible, their contribution must be bigger than the standard non-Gaussianities

A crude convention: $f_{\rm nl}$

The size of the non-Gaussianities are often described by a parameter $f_{\rm nl}$, defined by

$$\zeta(x) = \zeta_g(x) + f_{\rm nl}\,\left(\,\zeta_g{}^2(x) - \langle \zeta_g{}^2(x)\rangle\,\right)$$

Gaussian field

The three-point function is then

$$\langle 0\,|\,\zeta(x)\zeta(y)\zeta(z)\,|\,0\rangle = f_{\rm nl}\,\langle 0\,|\,\zeta_g{}^2(x)\zeta_g(y)\zeta_g(z)\,|\,0\rangle + \cdots$$

Calculating the non-Gaussianities this way,

$$\langle 0\,|\,\zeta(t,\mathrm{x})\zeta(t,\mathrm{y})\zeta(t,\mathrm{z})\,|\,0\rangle \propto \frac{f_{\rm nl}}{\varepsilon^2}\frac{H^4}{M_{\rm pl}{}^4}$$

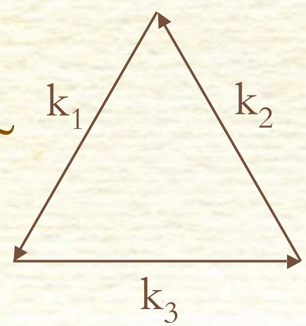
a slow-roll parameter

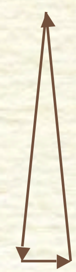
Two dimensions → two numbers

f_{nl} ignores much of the information in the three-point function

$$\langle 0(t)\,|\,\zeta(t,\mathrm{x})\,\zeta(t,\mathrm{y})\,\zeta(t,\mathrm{z})\,|\,0(t)\rangle$$
$$= \int \frac{d^3\mathrm{k}_1}{(2\pi)^3}\frac{d^3\mathrm{k}_2}{(2\pi)^3}\frac{d^3\mathrm{k}_3}{(2\pi)^3}\, e^{i\mathrm{k}_1\mathrm{x}} e^{i\mathrm{k}_2\mathrm{y}} e^{i\mathrm{k}_3\mathrm{z}}$$
$$(2\pi)^3\,\delta^3(\mathrm{k}_1+\mathrm{k}_2+\mathrm{k}_3)\,\frac{k_1^{\,3}+k_2^{\,3}+k_3^{\,3}}{k_1^{\,3}k_2^{\,3}k_3^{\,3}}\,A(k_1,k_2,k_3)$$

Sometimes the shape information is crudely captured by using f_{nl}'s for different triangles,

$f_{nl}^{\mathrm{equil}} \sim$ 

$f_{nl}^{\mathrm{local}} \sim$ 

Two dimensions → two numbers

$f_{\rm nl}$ ignores much of the information in the three-point function

$$\langle 0(t)\,|\,\zeta(t,\mathrm{x})\,\zeta(t,\mathrm{y})\,\zeta(t,\mathrm{z})\,|\,0(t)\rangle$$
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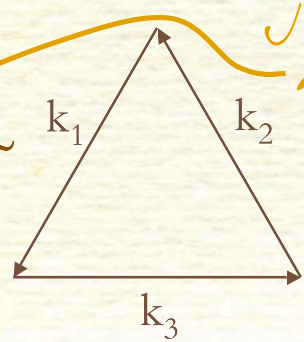
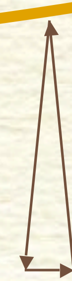
$$(2\pi)^3\,\delta^3(\mathrm{k}_1+\mathrm{k}_2+\mathrm{k}_3)\,\frac{k_1^{\,3}+k_2^{\,3}+k_3^{\,3}}{k_1^{\,3}k_2^{\,3}k_3^{\,3}}\,A(k_1,k_2,k_3)$$

2-dimensional function

Sometimes the shape information is crudely captured by using $f_{\rm nl}$'s for different triangles,

just two numbers

$f_{\rm nl}^{\rm equil} \sim$ 

$f_{\rm nl}^{\rm local} \sim$ 

Signals of Lorentz-breaking operators

So, what are the predictions for the $f_{\rm nl}$'s for the symmetry-breaking operators?

After a bit of calculating

[hep-ph/0905.4925]

Case I: $H^2\zeta^3$

$$|f_{\rm nl}| \sim \varepsilon^{1/2}\,\frac{M_{\rm pl}}{M}$$

Case II: $\zeta^2\,\vec{\nabla}\cdot\vec{\nabla}\zeta$

$$|f_{\rm nl}^{\rm local}| \ll |f_{\rm nl}^{\rm equil}| \sim \varepsilon^{1/2}\,\frac{M_{\rm pl}}{H}\,\frac{k}{k_\star}$$

$k_\star$ is a mode whose physical $k(t)$ equals M "initially"

$$\frac{H}{M} < \frac{k}{k_\star} < 1$$

Current & future bounds

Case I ($H^2 \zeta^3$): $$|f_{\rm nl}| \sim \varepsilon^{1/2} \frac{M_{\rm pl}}{M}$$

Case II ($\zeta^2 \vec{\nabla}\cdot\vec{\nabla}\zeta$): $$|f_{\rm nl}^{\rm equil}| \sim \varepsilon^{1/2} \frac{M_{\rm pl}}{H} \frac{k}{k_\star}$$

The "standard" inflationary prediction is that

$$f_{\rm nl} \sim O(\varepsilon)$$

The current WMAP-5 year limits are

$$-151 < f_{\rm nl}^{\rm equil} < 253$$
$$-9 < f_{\rm nl}^{\rm local} < 111$$

and PLANCK should reach $f_{\rm nl} \sim$ "a few"

Summary & comments

Compared with limits from the two-point function, the three-point function is more constraining,

Collins & Holman, Phys. Rev. D 77, 105016 (2008)

time-derivatives $\rightarrow$ $M > 10^{-2}\, M_{\rm pl}$

space derivatives $\rightarrow$ even more constraining

The reason is that the standard inflationary expectation is for *small* non-Gaussianities . . .

. . .so the signals of something new stand out all the more clearly.