Anna Kostouki, ***King's College London***

Inflation in first quantized String Cosmology from resummed in alpha' tachyon – dilaton backgrounds

JHEP 0903:022,2009 **(***e-Print:* ***arXiv:0811.4607*** *[hep-th]*)

J. Alexandre, A.K., N.E. Mavromatos

Invisible Universe **Conference, Paris, 1 July 2009**

This work was supported by the European Union through the FP6 Marie Curie Research and Training Network *UniverseNet* (MRTN-CT-2006-035863)

Outline

Motivation

String Inflation in 4 dimensions.

Our model

Closed Bosonic String in Graviton, Dilaton and Tachyon Backgrounds;
Field configurations that are resummed in the Regge slope, α'.

Conformal Properties

Conformal invariance conditions satisfied to all orders in the Regge slope, α'.

Cosmological Implications

FRW universe containing inflationary era (under conditions).

Motivation

Inflation:

• elegant & simple idea

• explains many cosmological observations (e.g. “horizon problem”, large - scale structure)

Inflation in String Theory:

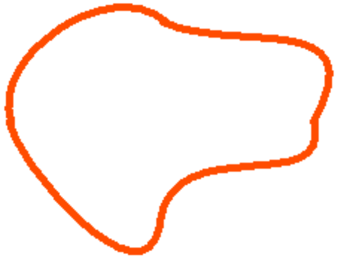
• effective theory

• in traditional string theories: compactification of extra dimensions of space-time is needed

• other models exist too, but no longer “simple & elegant”

Closed Bosonic String with Background fields

- Closed Bosonic String



D – dimensional "target" space – time

quantize string coordinates, $X^{\mu}(\tau, \sigma)$

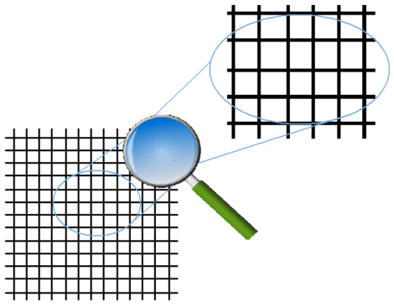
- Graviton, dilaton and tachyon background

$$g_{\mu\nu}(X) \quad \phi(X) \qquad T(X)$$

Target : Pass from the world-sheet quantum theory to the effective theory for the background fields

Tool : Conformal Invariance of the world-sheet theory

Physics in the target space – time is not affected by diffeomorphisms or rescaling of the world-sheet coordinates.



Conformal Invariance

Conformal invariance conditions:

$$\beta^g_{\mu\nu} = \beta^\phi = \beta^T = 0$$

Weyl anomaly coefficients must vanish.

They look like equations of motion for the background fields, i.e. one has to find background field configurations that satisfy these conditions.

$$\begin{aligned}
\beta^g_{\mu\nu} &= \alpha' R_{\mu\nu} + 2\alpha' \nabla_\mu \nabla_\nu \phi - \alpha' \partial_\mu T \partial_\nu T \\
\beta^\phi &= \frac{D-26}{6} - \frac{\alpha'}{2} \nabla^2 \phi + \alpha' \partial^\mu \phi \partial_\mu \phi \\
\beta^T &= -2T - \frac{\alpha'}{2} \nabla^2 T + \alpha' \partial^\mu \phi \partial_\mu T
\end{aligned}$$

Calculations of the Weyl anomaly coefficients are done perturbatively. The parameter in the perturbative expansion (equivalent to $\hbar$ in quantum mechanics) is the Regge slope, α'.

Field Configuration

We choose the following field configuration, which

- depends only on the time coordinate of the string, X^0 , and
- is non-perturbative in the Regge slope, α' :

$$
\begin{aligned}
g_{\mu\nu} &= \frac{A}{(X^0)^2}\eta_{\mu\nu} \\
\phi &= \phi_0 \ln\left(\frac{X^0}{\sqrt{\alpha'}}\right) \\
T &= \tau_0 \ln\left(\frac{X^0}{\sqrt{\alpha'}}\right)
\end{aligned}
$$

(A : constant with dimensions [mass]$^{-2}$
$\phi_0,\ \tau_0$:dimensionless constants)

$$\boxed{D = 4}$$

- **Study conformal properties** $\beta^g_{\mu\nu} = \beta^\phi = \beta^T \overset{?}{=} 0$
- **Study resulting cosmology**

Conformal properties

$$g_{\mu\nu} = \frac{A}{(X^0)^2}\eta_{\mu\nu}, \quad \phi = \phi_0 \ln\left(\frac{X^0}{\sqrt{\alpha'}}\right), \quad T = \tau_0 \ln\left(\frac{X^0}{\sqrt{\alpha'}}\right)$$

This configuration:

- doesn't satisfy the conformal invariance conditions at the "tree-level" (i.e. to first order in α'), but
- satisfies the conformal invariance conditions to all orders in α'.

This is true because:

(a) the definition of the Weyl anomaly coefficients is not unique, but is subject to *field redefinitions*, and

R. R. Metsaev and A. A. Tseytlin, Nucl. Phys. B 293 (1987) 385

(b) this configuration leads to Weyl anomaly coefficients with "*homogeneous*" dependence on X^0.

J. Alexandre, J. R. Ellis and N. E. Mavromatos, JHEP 0612 (2006) 071

Cosmological Properties

To study the cosmological implications of this model we need to study the space – time effective theory. The way to this effective theory is again through the conformal invariance conditions of the world-sheet theory.

Bosonic string with background fields; CFT on the world-sheet

$\Rightarrow$

Conformal Invariance Conditions

$$\beta^g_{\mu\nu} = \beta^\phi = \beta^T = 0$$

$\Downarrow$

E.o.m. for the fields in the target space – time

$\Leftarrow$

Effective action in the target space – time

Effective action

- The exact form of the effective action for closed strings with tachyon backgrounds, is not known.

- We therefore choose the most general two – derivative action (later we will place some constraints)

I. Swanson, Phys. Rev. D 78 (2008) 066020

$$
\begin{aligned}
S \quad = \quad \int d^D x \sqrt{-g} e^{-2\phi} \Big\{ \frac{D-26}{6\alpha'} + f_0(T) + f_1(T)R + 4 f_2(T) \partial_\mu \phi \partial^\mu \phi \\
- f_3(T) \partial_\mu T \partial^\mu T - f_4(T) \partial_\mu T \partial^\mu \phi \Big\}
\end{aligned}
$$

- Note that the choice of the functions $f_i(T)$ is not unique; redefinitions of the tachyon field will lead to different forms of these functions. The form of the scattering amplitudes is invariant under local field redefinitions and thus is not enough to determine these functions.

Effective action

We distinguish two “frames” for the effective space – time theory.

- We started with the “*Sigma–model frame*”, where the Einstein – Hilbert term takes the form:

$$S \sim \int d^D x \sqrt{-g}\, e^{-2\phi} f_1(T) \Big\{ R + \dots \Big\}$$

- We need to pass to the *“Einstein frame”*, where the Einstein – Hilbert term takes its canonical form:

$$S^E \sim \int d^D x \sqrt{-g^E} \Big\{ R^E + \dots \Big\}$$

I. Antoniadis, C. Bachas, J. R. Ellis, D. V. Nanopoulos, Nucl. Phys. B 328(1989) 117

This is done through a metric redefinition :

$$\begin{aligned} g_{\mu\nu} \to g^E_{\mu\nu} \; &= \; e^{\omega(\phi,T)} g_{\mu\nu} \\ \omega(\phi,T) \; &= \; \frac{-4\phi + 2\ln f_1(T)}{D-2} \end{aligned}$$

Sigma-model frame

Our metric configuration is: $g_{\mu\nu} = \frac{A}{(x^0)^2}\eta_{\mu\nu}$

This configuration gives INFLATION in the Sigma-model frame:

$$ds^2 = \frac{A}{(x^0)^2}\left[(dx^0)^2 - (d\mathbf{x})^2\right]$$

$$\left.\begin{aligned} y^0 &= -\sqrt{A}\ln\frac{x^0}{\sqrt{\alpha'}} \\ x^i &\rightarrow \sqrt{\frac{A}{\alpha'}}x^i \end{aligned}\right\}$$

$$ds^2 = (dy^0)^2 - e^{2y^0/\sqrt{A}}(d\mathbf{x})^2$$

$$\boxed{\begin{aligned} a(y^0) &= e^{y^0/\sqrt{A}} \\ H &= \frac{1}{\sqrt{A}} \end{aligned}}$$

Einstein Frame

1st choice for : $f_1(T) = e^{-T}$

A. A. Tseytlin, J. Math. Phys. 42 (2001) 2854

If $2\phi_0 + \tau_0 = 0$ → **Inflation**

If $2\phi_0 + \tau_0 \neq 0$ → **Power-law expansion**

Exit from "eternal" inflation?

Is there a way to exit the "eternal" inflation era, and enter a power-law expansion (or even Minkowski) era?

- The condition $2\phi_0 + \tau_0 = 0$ has to be disturbed.

  for example: (yet unknown) mechanism which makes the tachyon field decay to zero

- Different choices for f_1 will give different cosmologies.

  We exploit solutions that correspond to choices for $f_1(T)$ that are resummations of exponentials of T. These solutions contain inflationary periods for specific eras.

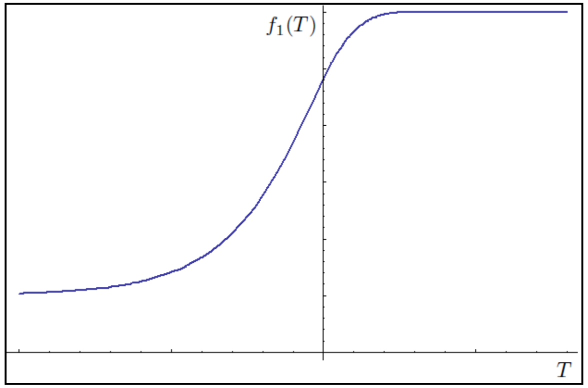
Solutions containing Inflationary eras

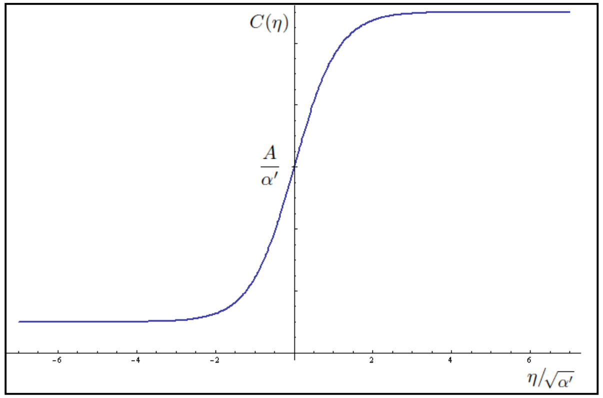
2nd choice for f_1 :

$$f_1(T) = 1 + B\tanh\left(\sqrt{\alpha'}\rho e^{T/\tau}\right)$$

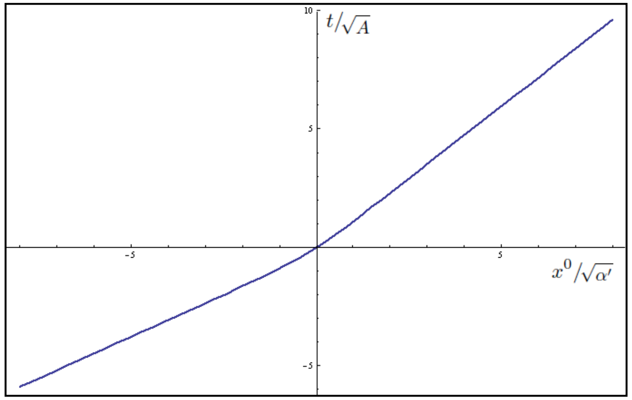
$$\phi_0 = -1,\ \tau_0 = \tau$$

$$C(\eta) = \frac{A}{\alpha'}\left[1 + B\tanh(\rho\eta)\right]$$

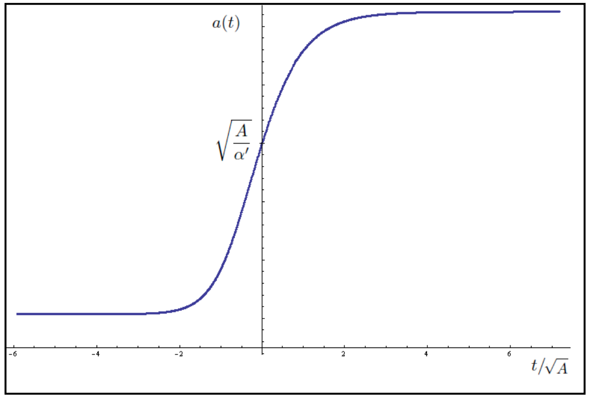




Conformal
Scale Factor



Flow of cosmic time with
sigma-model frame time



Scale Factor
in the Einstein frame

S. S. Gubser, Phys. Rev. D 69 (2004) 123507

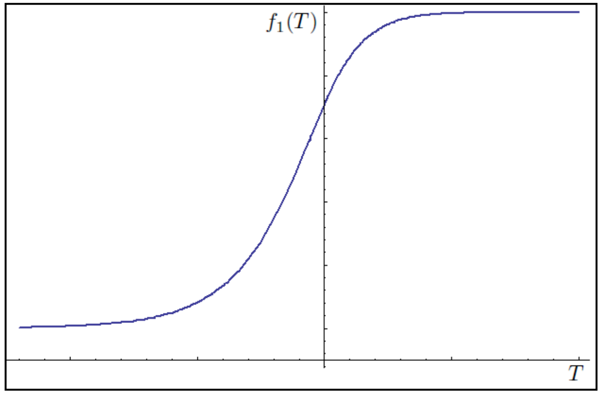
Solutions containing Inflationary eras

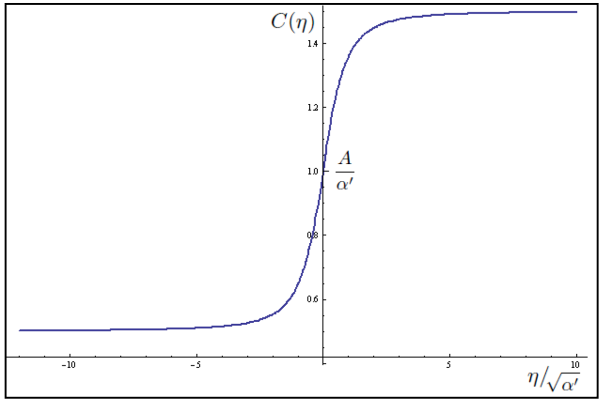
3rd choice for f_1 :

$$f_1(T) = 1 + B\frac{e^{T/\tau}}{\sqrt{e^{2T/\tau} + \rho^2/\alpha'}}$$

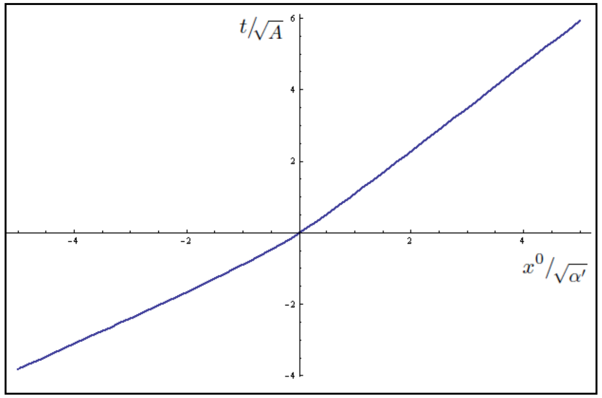
$$\phi_0 = -1, \ \tau_0 = \tau$$

$$C(\eta) = \frac{A}{\alpha'}\left(1 + B\frac{\eta}{\eta^2 + \rho^2}\right)$$

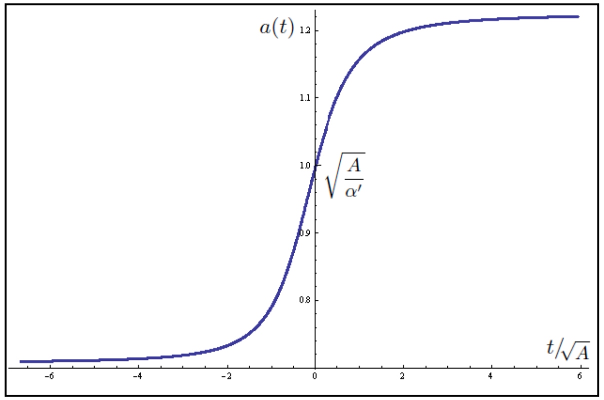




Conformal
Scale Factor



Flow of cosmic time with
sigma-model frame time



Scale Factor
in the Einstein frame

S. S. Gubser, Phys. Rev. D 69 (2004) 123507

Comments

1. This approach is based on perturbative String Theory.

   i. The string coupling has to be small. (Remember: $g_s = e^{\phi}$)

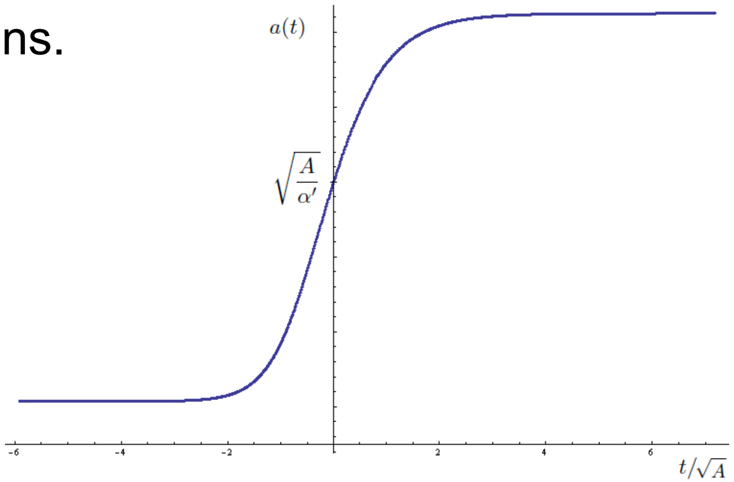
   We checked that for all our solutions we get:

   $$g_s \;=\; e^{\phi} \;\xrightarrow{t\longrightarrow\infty}\; 0$$

   ii. String amplitudes have to be well defined $\leftrightarrow$

   The Universe must be free of horizons.

   We choose this solution to be self – consistent within our approach



Comments

2 What about the Tachyons? Is this model stable?

i There shouldn't be any "ghost" fields, i.e. fields whose kinetic terms have the wrong sign in the effective action. This refers to both the tachyon and the dilaton.

ii We would ideally want the tachyon potential in the effective action to go to zero at late times, so that the tachyons don't have any effect at late times.

To be sure that these requirements are satisfied, one should know the exact form of the effective action. However, we found that placing some mild constraints on the functions that appear in it (e.g. they don't diverge at late times), they are both satisfied.

Summary & Conclusions

1. Started from an α'-resummed configuration for graviton, dilaton and tachyon backgrounds of a closed bosonic String Theory.

   Showed that it is consistent with conformal invariance conditions to all oders in α'.

2. Studied the Cosmological implications of this model (in the Einstein frame) and found that a de-Sitter Universe, as well as a power-law expanding universe (that can be chosen to be free of horizons) is possible.

   Also found cosmological solutions that interpolate between an Inflationary era and Minkowski universes. Since our approach is based on String scattering amplitudes, it is these solutions that are self-consistent configurations.

Summary & Conclusions

3. All our solutions may be realized in 4 dimensions, thanks to the presence of the background fields.

4. Commented on some general consistency and stability checks of the model.

Thank you for your attention !

SPARE SLIDES

References:

- *J. Alexandre, J. R. Ellis and N. E. Mavromatos*, JHEP 0612 (2006) 071 [hep-th/0610072]
- *I. Swanson*, Phys. Rev. D 78 (2008) 066020 [hep-th/0804.2262]
- *R. R. Metsaev and A. A. Tseytlin*, Nucl. Phys. B 293 (1987) 385
- *A. A. Tseytlin*, J. Math. Phys. 42 (2001) 2854 [hep-th/0011033]
- *S. S. Gubser*, Phys. Rev. D 69 (2004) 123507 [hep-th/0305099]
- *J. Schnittger and U. Ellwanger*, Theor. Math. Phys. 95 (1993) 643 [hep-th/9211139]
- *I. Antoniadis, C. Bachas, J. R. Ellis, D. V. Nanopoulos*, Nucl. Phys. B 328(1989) 117

One solution to the Conformal Invariance conditions is the following:

$$\begin{aligned} g_{\mu\nu} &= \eta_{\mu\nu} \\ \phi &= \mathrm{const} \\ T &= 0 \end{aligned}$$

NB. This is a perturbative solution. It satisfies the conformal invariance conditions only to 1st order in α'.

Conformal properties

General field redefinition: $g^i \;\longrightarrow\; \tilde{g}^i \equiv g^i + \delta g^i$

- Theory is invariant
- Weyl anomaly coefficients transform:

$$
\begin{aligned}
\beta^i \quad &\longrightarrow \quad \tilde{\beta}^i \\
\tilde{\beta}^i \quad &= \quad \beta^i + \delta g^j \frac{\delta \beta^i}{\delta g^j} - \beta^j \frac{\delta(\delta g^i)}{\delta g^j}
\end{aligned}
$$

$$
\left(
\begin{aligned}
g^i \;&=\; (\; g_{\mu\nu},\; \phi,\; T \;) \\
\beta^i \;&=\; (\; \beta^g_{\mu\nu},\; \beta^\phi,\; \beta^T \;)
\end{aligned}
\right)
$$

Conformal properties

"*Homogeneous*" dependence on X^0

$$\beta^g_{\mu\nu} = \frac{E_{\mu\nu}}{(X^0)^2} + \mathcal{O}\left(\alpha'^3\right)$$

$$\beta^\phi = E_1 + \mathcal{O}\left(\alpha'^3\right)$$

All this is due to the specific form of our configuration **!**

where

$$E_{\mu\nu}, E_1 = \text{ constants which contain factors of } \alpha' \text{ up to } \alpha'^2$$

Power counting → Every other term that appears at higher loops in the beta-functions is homogeneous

[Ellwanger, Schnittger 1992]

Conformal properties

"*Homogeneous*" dependence on X^0

$$\beta^g_{\mu\nu} = \frac{E_{\mu\nu}}{(X^0)^2} + \mathcal{O}\left(\alpha'^3\right)$$

$$\beta^\phi = E_1 + \mathcal{O}\left(\alpha'^3\right)$$

$$\beta^T = -2T + E_2 + \mathcal{O}\left(\alpha'^3\right)$$

$$-2\tau_0 \ln \frac{X^0}{\sqrt{\alpha'}}$$

"inhomogeneous term"

constant

Power counting → Every other term that appears at higher loops is a constant

Conformal properties

Based on this "homogeneity" property

(of all, besides one, terms in the Weyl anomaly coefficients),

we can use the freedom of general field redefinitions to prove

that the Weyl anomaly coefficients vanish to arbitrary order.

Conformal properties

Starting from the two – loop Weyl anomaly coefficients,

This analysis can be repeated order – by – order to all orders

This configuration satisfies

Conformal Invariance to all orders in α_

$$g_{\mu\nu} = \frac{A}{(X^0)^2}\eta_{\mu\nu}, \quad \phi = \phi_0 \ln\left(\frac{X^0}{\sqrt{\alpha'}}\right), \quad T = \tau_0 \ln\left(\frac{X^0}{\sqrt{\alpha'}}\right)$$

Conformal properties

One can find a general field redefinition, that:

1. Doesn't change the X^0 - dependence of the fields
2. Cancels the inhomogeneous term in β^T
3. Only adds new homogeneous terms to the Weyl anomaly coefficients

→ Leads to new Weyl anomaly coefficients:

$$\beta^g_{\mu\nu} = \frac{\tilde{E}_{\mu\nu}}{(X^0)^2} + \mathcal{O}\left(\alpha'^3\right)$$

$$\beta^\phi = \tilde{E}_1 + \mathcal{O}\left(\alpha'^3\right)$$

$$\beta^T = \tilde{E}_2 + \mathcal{O}\left(\alpha'^3\right)$$

Enough freedom in the parameters of the redefinition to make:

$$\tilde{E}_{\mu\nu} = \tilde{E}_1 = \tilde{E}_2 = 0$$

$$\Downarrow$$

$$\beta^g_{\mu\nu} = \beta^\phi = \beta^T = 0$$

up to $\mathcal{O}\left(\alpha'^2\right)$

Einstein Frame

$$ds^2 \equiv dt^2 - a^2(t)d\mathbf{r}^2 = \frac{A\, e^{\omega}}{(x^0)^2}\left[(dx^0)^2 - (d\mathbf{x})^2\right]$$

$$\omega\,(\phi, T) = \frac{-4\phi + 2\ln f_1(T)}{D-2}$$

Note: if $\omega(\phi, T) = 0$, then the two frames coincide

Cosmology can be derived from here ...

$$t = t(x^0) = \ldots$$
$$a(t) = \ldots$$

Important role:

- Form of the function $f_1(T)$
- Form of the configuration $\phi(x^0),\ T(x^0)$

Our configuration is: $\phi = \phi_0 \ln \frac{x^0}{\sqrt{\alpha'}},\ T = \tau_0 \ln \frac{x^0}{\sqrt{\alpha'}}$

Einstein Frame: "Eternal" Inflation

1st choice for f_1 : $f_1(T) = e^{-T}$ ***[Tseytlin, 2001]***

If $2\phi_0 + \tau_0 = 0$ → Einstein frame ≡ Sigma – model frame

- Cosmic time: $t \propto -\sqrt{A}\ln\frac{x^0}{\sqrt{\alpha'}}$

- Scale factor: $a(t) = a_0 \exp\left(\frac{t}{\sqrt{A}}\right)$

Inflation

$$H = \frac{1}{\sqrt{A}}$$

Einstein Frame: Power-law expansion

1st choice for f_1 : $f_1(T) = e^{-T}$ ***[Tseytlin, 2001]***

If $2\phi_0 + \tau_0 \neq 0$ → Einstein frame ≠ Sigma – model frame

- Cosmic time: $t \propto \left(\frac{x^0}{\sqrt{\alpha'}}\right)^{-\frac{2\phi_0+\tau_0}{D-2}}$

Power-law expansion

- Scale factor: $a(t) \propto t^{1+\frac{D-2}{2}\left(\phi_0+\frac{\tau_0}{2}\right)^{-1}}$

$2\phi_0 + \tau_0 > 0$
$\longrightarrow$ no horizons

Final remarks

Consistency and stability issues:

- String coupling ? We want: $$g_s \;=\; e^{\phi} \;\xrightarrow{t\to\infty}\; 0$$

$$f_1(T) \;=\; e^{-T} \;\rightarrow\; g_s \;=\; e^{\phi} \;\left.\begin{array}{ll} \propto\; -\phi_0\, t\;, & 2\phi_0+\tau_0=0 \\ \propto\; t^{-\frac{D-2}{2\phi_0+\tau_0}\phi_0}\;, & 2\phi_0+\tau_0\neq 0 \end{array}\right\} \Rightarrow\quad \phi_0>0\;!$$

$$f_1(T) = \ldots \quad\rightarrow\quad g_s \;\propto\; t^{\phi_0} \quad\Rightarrow\quad \phi_0<0$$ (our choice: $\phi_0=-1$, ok)

(2nd & 3rd case)

- Ghost fields? Dilaton and Tachyon fields appear with the wrong sign in the Sigma-model frame action. When passing to the Einstein frame and after rediagonalizing the action, we can place certain (mild) constraints on the functions $f_i(T)$ that can guarantee that the kinetic terms for both fields have the right sign.

Final remarks

The constraints that we have to place, in the most general case, are:

$$f_2 + \frac{[4(D-1)f_1' - (D-2)f_4]^2}{8(D-2)\left[(D-1)\frac{(f_1')^2}{f_1} + (D-2)f_3\right]} < \frac{D-1}{D-2}f_1$$

$$f_3 + \frac{D-1}{D-2}\frac{(f_1')^2}{f_1} > 0$$

For the interpolating solution case, these become even milder, as the effective action at late times takes the form:

$$S^E_{\text{late times}} \sim \int d^D x\sqrt{-g}\left\{ R + e^{\frac{4\phi}{D-2}}\left[\frac{D-26}{6\alpha'} + f_0(T)\right] - \left[\frac{4(D-1)}{D-2} - 4f_2(T)\right]\partial\phi\cdot\partial\phi \right.$$
$$\left. - f_3(T)\partial T\cdot\partial T - f_4(T)\partial\phi\cdot\partial T \right\}$$

All the kinetic terms fall of as t^{-2} at late times, so as long as f_2 , f_3 and f_4 don't diverge faster than this, the kinetic terms for both fields effectively disappear from the action at late times.

Final remarks

- Tachyon Potential? In our model we haven't considered any specific form for the tachyon potential. In the Einstein frame action it takes the form:

$$e^{\frac{4\phi}{D-2}} [f_1(T)]^{-\frac{D}{D-2}} \left[\frac{D-26}{6\alpha'} + f_0(T) \right]$$

For our interpolating solutions, as long as $f_0(T)$ doesn't diverge, this falls asymptotically to zero:

$$e^{\frac{4\phi}{D-2}} [f_1(T)]^{-\frac{D}{D-2}} \left[\frac{D-26}{6\alpha'} + f_0(T) \right] \sim t^{-\frac{4}{D-2}} \left[\frac{D-26}{6\alpha'} + f_0(T) \right]$$

For our power-law expanding solutions, a similar relation is true as long as the tachyon amplitude is chosen to be greater than zero, $\tau_0 > 0$ (which is consistent with all the constraints that we have placed so far on it).