Gravastars or Black Holes as Consequence of the Einstein's Theory of Gravity

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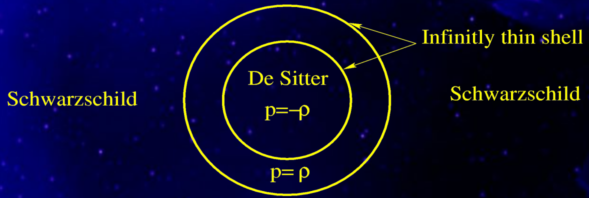

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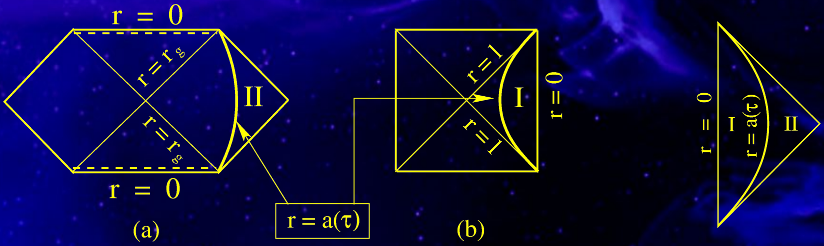

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- **"Bounded excursion" gravastars:**
As VW noticed, there is a less stringent notion of stability, the so-called "bounded excursion" models, in which there exist two radii a_1 and a_2 such that
$V\left(a_1\right) = 0, \quad V'\left(a_1\right) \leq 0, \quad V\left(a_2\right) = 0, \quad V'\left(a_2\right) \geq 0$, with $V(a) < 0$ for $a \in \left(a_1, a_2\right)$, where $a_2 > a_1$.

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- **In previous works we studied the gravastars models proposed by VW (JCAP 6, 25 (2008), JCAP 11, 10 (2008)), and found that, among other things, such configurations can indeed be constructed, although the region for the formation of these types of gravastars is very small in comparison to that of black holes.**

- *Here, we generalize theses results to the case where the equation of state of the infinitely thin shell is given by $p = (1-\gamma)\sigma$ with γ being a constant, with an interior phantom/standard energy fluid (JCAP 03,10 (2009)).*

- *Here, we generalize theses results to the case where the equation of state of the infinitely thin shell is given by* $p = (1-\gamma)\sigma$ *with* γ *being a constant, with an interior phantom/standard energy fluid (JCAP 03,10 (2009)).*
- *We constructed three-layer dynamical models, in analogy to the VW models, and then, we show that both types of stable gravastars and black holes exist for several situations. However, we do not find a configuration with a pure dark energy in the interior. In the phase space, the region of gravastars and of black holes are non-zero.*

Dynamical Three-layer Prototype Gravastars

The interior: anisotropic dark energy fluid (Lobo's model CQG 23, 1525 (2006)) with a metric given by

$$ds_-^2 = -f_1 dt^2 + f_2 dr^2 + r^2 d\Omega^2,$$

where $d\Omega^2 \equiv d\theta^2 + \sin^2(\theta) d\phi^2$, and

$$f_1 = (1 + br^2)^{\frac{1-\omega}{2}} (1 + 2br^2)^{\omega}, \qquad f_2 = \frac{1 + 2br^2}{1 + br^2},$$

with $\bar{m}(r) = \frac{br^3}{2(1+2br^2)}$

The corresponding energy density ρ, radial and tangential pressures p_r and p_t are given, respectively, by

$$\begin{aligned} p_r &= \omega\rho = \left(\frac{\omega b}{8\pi}\right)\left(\frac{3+2br^2}{(1+2br^2)^2}\right), \\ p_t &= -\omega\rho + b^2 r^2 \left\{(1+\omega)(3+2br^2)\left[(1+3\omega)+2br^2(1+\omega)\right] \right. \\ & \left. -8\omega(5+2br^2)(1+br^2)\right\} \left\{32\pi\left[(1+2br^2)^3(1+br^2)\right]\right\}. \end{aligned}$$

- **The exterior:** spacetime is given by the Schwarzschild metric

$$ds_+^2 = -f dv^2 + f^{-1} d\mathbf{r}^2 + \mathbf{r}^2 d\Omega^2,$$

where $f = 1 - 2m/\mathbf{r}$.

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- The thin shell: the hypersurface divided these two spacetimes is given by the metric

$$ds_\Sigma^2 = -d\tau^2 + R^2(\tau) d\Omega^2.$$

Since $\left(ds_-^2\right)_\Sigma = \left(ds_+^2\right)_\Sigma = ds_\Sigma^2$, we find that $r_\Sigma = \mathbf{r}_\Sigma = R$, and

$$f_1 \dot{t}^2 - f_2 \dot{R}^2 = 1, \qquad f \dot{v}^2 - \frac{\dot{R}^2}{f} = 1.$$

The interior and exterior extrinsic curvature are given by

$$
\begin{aligned}
K^{-}_{\tau\tau} &= \frac{1}{2}(1+bR^2)^{-\omega/2}\dot{t}\left\{\left[4(1+bR^2)^{\omega/2}bR^2\dot{R}^2\right.\right. \\
&+2(1+bR^2)^{\omega/2}\dot{R}^2-(1+2bR^2)^{\omega}\sqrt{1+bR^2}\,bR^2\dot{t}^2 \\
&\left.-(1+2bR^2)^{\omega}\sqrt{1+bR^2}\,\dot{t}^2\right](2bR^2\omega+2bR^2+3\omega+1) \\
&\left.-2(1+bR^2)^{\omega/2}(1+2bR^2)\dot{R}^2\right\} \\
&(1+2bR^2)^{-2}(1+bR^2)^{-1}bR+\dot{R}\ddot{t}-\ddot{R}\dot{t}, \\
K^{-}_{\theta\theta} &= \frac{\dot{t}(1+bR^2)R}{1+2bR^2}, \quad K^{-}_{\phi\phi}=K^{-}_{\theta\theta}\sin^2(\theta), \\
K^{+}_{\tau\tau} &= \dot{v}\frac{m(4m^2\dot{v}^2-4mR\dot{v}^2-3R^2\dot{R}^2+R^2\dot{v}^2)}{(2m-R)R^3}+\dot{R}\ddot{v}-\ddot{R}\dot{v}, \\
K^{+}_{\theta\theta} &= -\dot{v}(2m-R), \quad K^{+}_{\phi\phi}=K^{+}_{\theta\theta}\sin^2(\theta).
\end{aligned}
$$

- From the Lake's work (PRD 19, 2847 (1979)) we have

$\Rightarrow \quad [K_{\theta\theta}] = K^{+}_{\theta\theta} - K^{-}_{\theta\theta} = -M.$

which furnishes

$$M = -R\sqrt{1 - \frac{2m}{R} + \dot{R}^2} + R\frac{\left[1 + bR^2 + \dot{R}^2(1 + 2bR^2)\right]^{1/2}}{(1 + bR^2)^{-(\omega+1)/4}(1 + 2bR^2)^{\frac{\omega+2}{2}}},$$

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- and

$$\dot{M} + 8\pi R\dot{R}\vartheta = 4\pi R^2[T_{\alpha\beta}u^{\alpha}n^{\beta}] = \pi R^2\left(T^{+}_{\alpha\beta}u^{\alpha}_{+}n^{\beta}_{+} - T^{-}_{\alpha\beta}u^{\alpha}_{-}n^{\beta}_{-}\right),$$

which reduces to
$\dot{M} + 8\pi R\dot{R}(1-\gamma)\sigma = 0 \Rightarrow M = kR^{2(\gamma-1)}$, if we assume the equation of state given by $\vartheta = (1-\gamma)\sigma$.

Recall that $\frac{1}{2}\dot{R}^2 + V(R) = 0 \quad \Rightarrow \quad V(R) = -\frac{1}{2}\dot{R}^2$, we find

$$
\begin{aligned}
V(R, m, \omega, b, \gamma) = &-\Big\{ b_2^{(\omega+2)} R^{4(\gamma-1)} b_1^{(\omega+1)/2} \\
&-2b_2^{(3\omega+4)/2} R^{2(\gamma-1)} b_1^{(\omega+1)/4} \Big[b_2^{(-\omega)} b_1^{(\omega+1)/2} R^2 \\
&-b_2^{-(\omega+1)} b_1^{(\omega+3)/2} R^2 - 2b_2^{(-\omega)} b_1^{(\omega+1)/2} mR + b_1 R^2 + b_2 R^2 \\
&+2b_2 mR + b_2 R^{4(\gamma-1)} \Big]^{1/2} + b_2^{(\omega+2)} R^2 b_1^{(\omega+1)/2} \\
&-b_2^{(2\omega+3)} R^2 - 2b_2^{(\omega+2)} mR b_1^{(\omega+1)/2} \\
&+2b_2^{(2\omega+3)} mR + b_2^{(2\omega+3)} R^{4(\gamma-1)} - b_1^{(\omega+2)} R^2 \\
&+b_2^{(\omega+1)} b_1^{(\omega+3)/2} R^2 \Big\} \Big/ \left\{ 2R^2 b_2 \left[b_2^{(\omega+1)} - b_1^{(\omega+1)/2} \right]^2 \right\}.
\end{aligned}
$$

where $b_1 \equiv 1 + bR^2$ and $b_2 \equiv 1 + 2bR^2$, and we have rescalled m, b, R as $m \longrightarrow mk^{-\frac{1}{2\gamma-3}}$, $\quad b \longrightarrow bk^{\frac{2}{2\gamma-3}}$, $\quad R \longrightarrow Rk^{-\frac{1}{2\gamma-3}}$.

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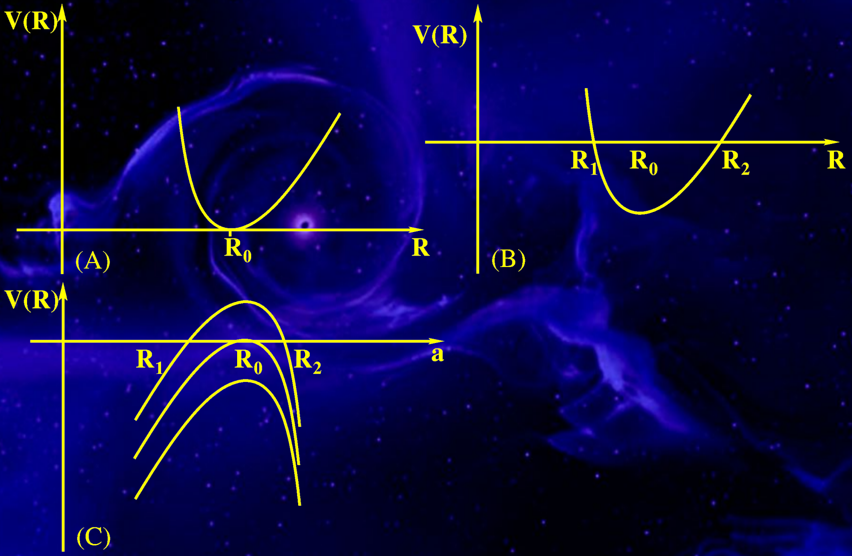

Clearly, for any given constants m, ω, b and γ, the potential $V(R)$ uniquely determines the collapse of the prototype gravastar. Depending on the initial value R_0, the collapse can form either a black hole, a gravastar, a Minkowski, or a spacetime filled with phantom fluid.

To avoid any kind of initial horizons, cosmological or event, we impose $R_0 > 2m$.

Since the potential $V(R)$ is so complicated, it is too difficult to study it analytically. Instead, in the following we shall study it numerically. Before doing so, we shall present the classifications of matter as dark energy or phantom energy for anisotropic interior fluid. Taking several values of ω in the intervals $-1 < \omega < -1/3$ and $\omega > 0$, we could not found any case where the interior dark energy exist.

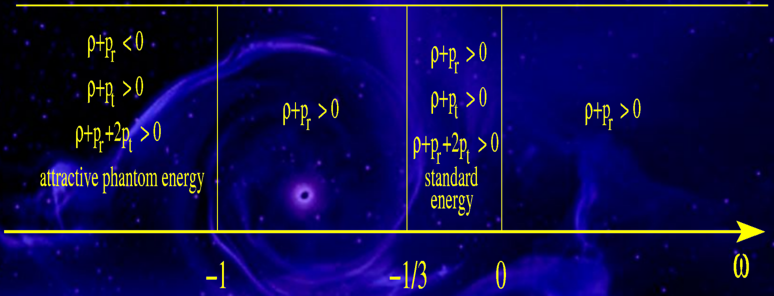

Fig 4: In this figure we show the intervals of ω for which the weak and strong energy conditions are independent of the coordinate R and the parameter b. The condition $\rho + p_r > 0$ is violated for $\omega < -1$, while the conditions $\rho + p_t > 0$ and $\rho + p_r + 2p_t > 0$ are satisfied for $\omega < -1$ and $-1/3 < \omega < 0$, for any values of R and b. For the others intervals of ω the analysis of the energy conditions depends on a complex relation of R and b.

In order to fulfill the energy condition $\sigma + 2p \geq 0$ of the shell and assuming that $p = (1 - \gamma)\sigma$ we must have $\gamma \leq 3/2$. On the other hand, in order to satisfy the condition $\sigma + p \geq 0$, we obtain $\gamma \leq 2$. Hereinafter, we will use only some particular values of the parameter γ which are analyzed in this work.

Table: This table summarizes the classification of matter on the thin shell, based on the energy conditions. The last column indicates the particular values of the parameter γ, where we assume that $\sigma \geq 0$.

Matter	Condition 1	Condition 2	γ
Normal Matter	$\sigma + 2p \geq 0$	$\sigma + p \geq 0$	-1 or 0
Dark Energy	$\sigma + 2p < 0$	$\sigma + p \geq 0$	7/4
R. Ph. Energy	$\sigma + 2p < 0$	$\sigma + p < 0$	3
A. Ph. Energy	$\sigma + 2p \geq 0$	$\sigma + p < 0$	Not possible

In the following we will discuss three physical possibilities for the type of system that can be formed from the study of the potential $V(R, m, \omega, b, \gamma)$:

- (a) Black hole or dispersion of the matter,

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- (b) Gravastar or normal star,
- (c) Black hole or phantom gravastar.

Black Hole or Dispersion of the Matter

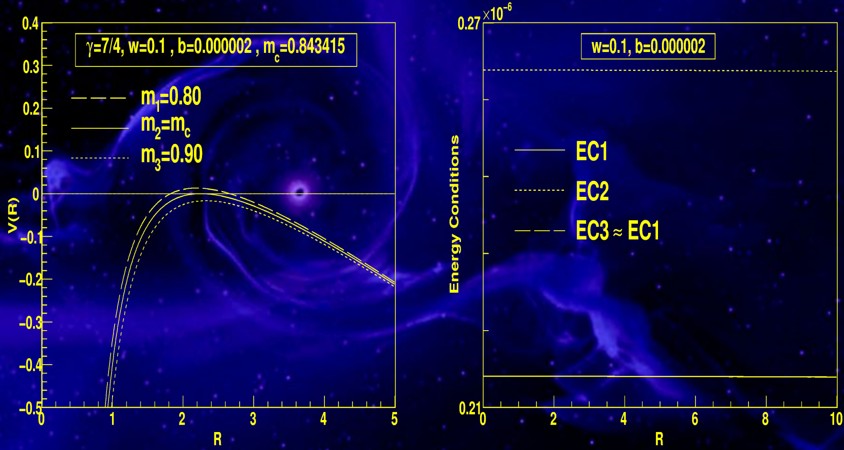
For $m > m_c$ the potential $V(R)$ is strictly negative. Then, the collapse always forms black holes. For $m = m_c$, there are two different possibilities, depending on the choice of the initial radius R_0. In particular, if the star begins to collapse with $R_0 > R_c$, it will approach to the minimal radius R_c. Once it reaches this point, the shell will stop collapsing. However, this point is unstable and any small perturbations will lead the star either to expand for ever and leave behind a flat spacetime, or to collapse until $R = 0$, whereby a Schwarzschild black hole is finally formed. On the other hand, if the star begins to collapse with $2m_c < R_0 < R_c$ as shown in the following figures, the star will collapse until a black hole is formed. For $m < m_c$, the potentials $V(R)$ for each case have a positive maximal, and the equation $V(R, m < m_c) = 0$ has two positive roots $R_{1,2}$ with $R_2 > R_1 > 0$.

For these cases there are also two possibilities, depending on the choice of the initial radius R_0:

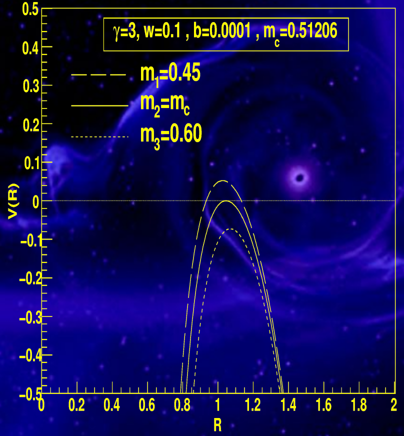
- If $R_0 > R_2$, the star will first contract to its minimal radius $R = R_2$ and then expand to infinity, whereby a spacetime fulfilled by phantom energy fluid is finally formed.

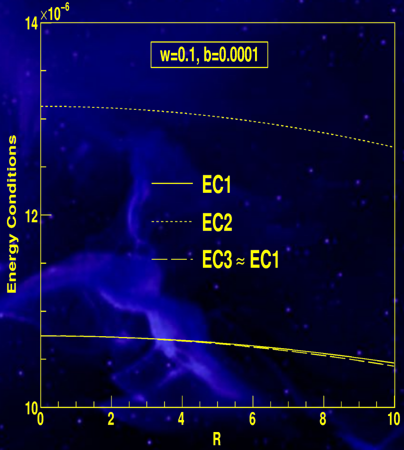
For these cases there are also two possibilities, depending on the choice of the initial radius R_0:

- If $R_0 > R_2$, the star will first contract to its minimal radius $R = R_2$ and then expand to infinity, whereby a spacetime fulfilled by phantom energy fluid is finally formed.
- If $2m < R_0 < R_1$, the star will collapse continuously until $R = 0$, and a black hole will be finally formed.

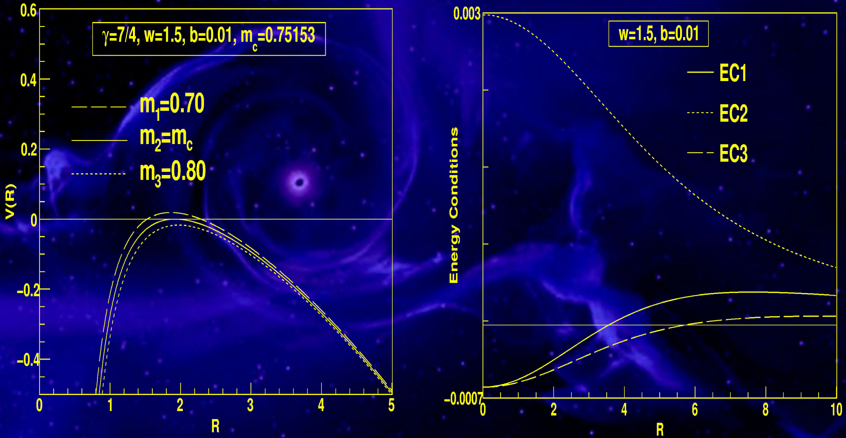


Case B: The potential $V(R)$ and the energy conditions EC1$\equiv \rho + p_r + 2p_t$, EC2$\equiv \rho + p_r$ and EC3$\equiv \rho + p_t$

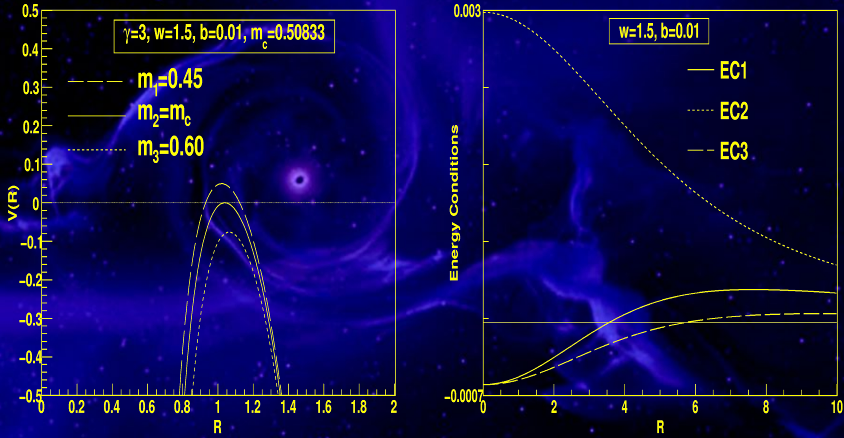




Case C: The potential $V(R)$ and the energy conditions EC1$\equiv \rho + p_r + 2p_t$, EC2$\equiv \rho + p_r$ and EC3$\equiv \rho + p_t$



Case H: The potential $V(R)$ and the energy conditions EC1$\equiv \rho + p_r + 2p_t$, EC2$\equiv \rho + p_r$ and EC3$\equiv \rho + p_t$



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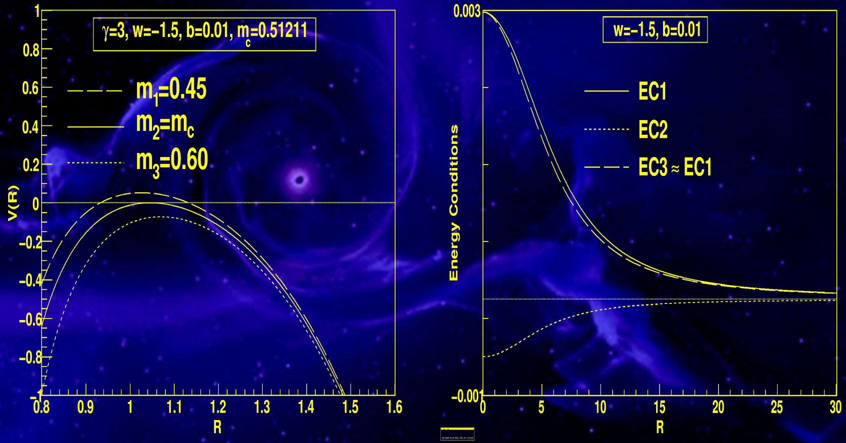
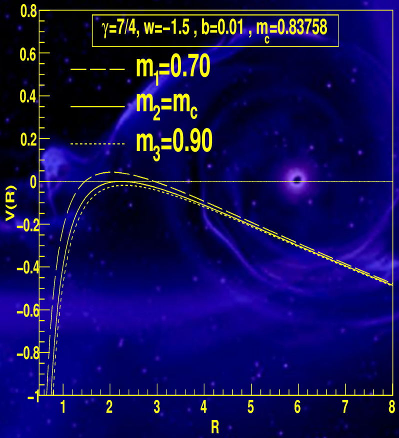
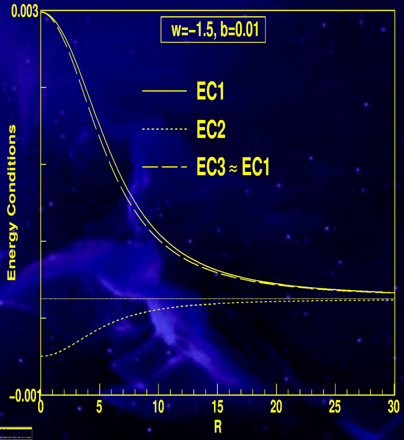


Case K: The potential $V(R)$ and the energy conditions EC1$\equiv \rho + p_r + 2p_t$, EC2$\equiv \rho + p_r$ and EC3$\equiv \rho + p_t$



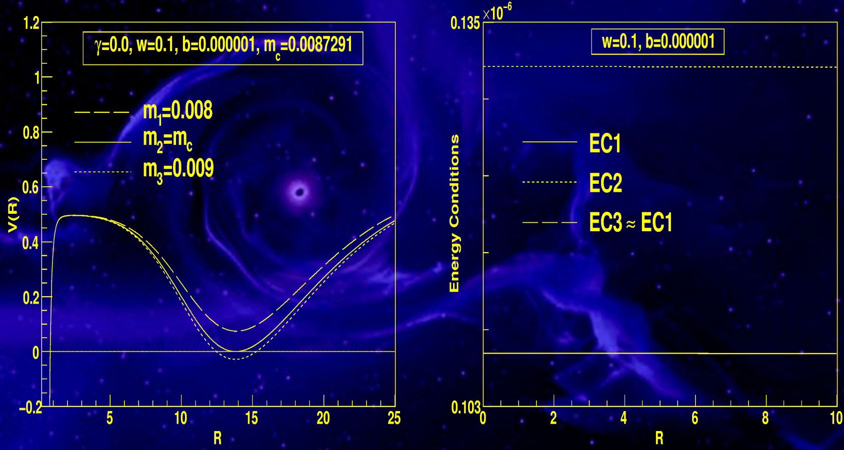


Case L:The potential $V(R)$ and the energy conditions EC1$\equiv \rho + p_r + 2p_t$, EC2$\equiv \rho + p_r$ and EC3$\equiv \rho + p_t$

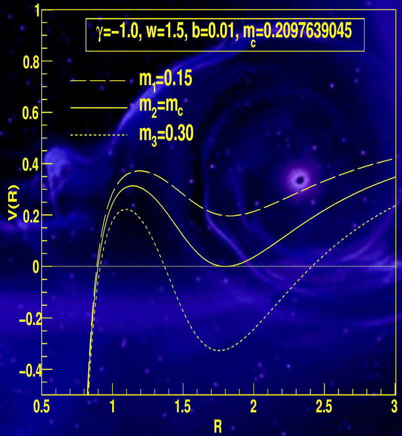
Gravastar or Normal Star

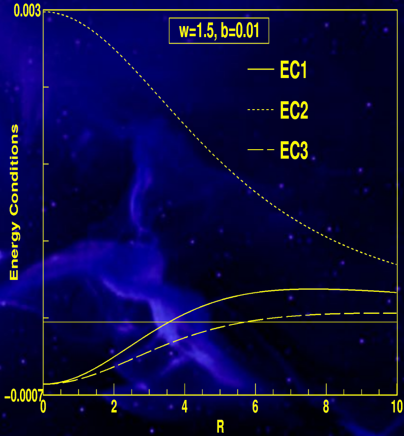
In this case we can see that $V(R) = 0$ now can have one, two or three real roots, depending on the mass of the shell. For $m > m_c$ we have, say, R_i, where $R_{i+1} > R_i$. If we choose $R_0 > R_3$ (for $m = m_c$ we have $R_2 = R_3$), then the star will not be allowed in this region because the potential is greater than the zero. However, if we choose $R_1 < R_0 < R_2$, the collapse will bounce back and forth between $R = R_1$ and $R = R_2$. Such a possibility is shown in the following figures. This is exactly the so-called **"bounded excursion"** model mentioned in VW, and studied in some details in two previous works of ours. Of course, in a realistic situation, the star will emit both gravitational waves and particles, and the potential shall be self-adjusted to produce a minimum at $R = R_{static}$ where $V\left(R = R_{static}\right) = 0 = V'\left(R = R_{static}\right)$ whereby a gravastar or a normal star is finally formed, although we have found, in that previous works, that for the gravastar with De Sitter's interior, the potential tends to $-\infty$ when R tends to ∞.

Here it is completely different since the potential now tends to $+\infty$ when R tends to ∞. Thus, in the cases studied here we do not have situations where the star expands leaving behind a flat spacetime, as in (JCAP 6, 25 (2008), JCAP 11, 10 (2008)).



Case A: The potential $V(R)$ and the energy conditions EC1$\equiv \rho + p_r + 2p_t$, EC2$\equiv \rho + p_r$ and EC3$\equiv \rho + p_t$

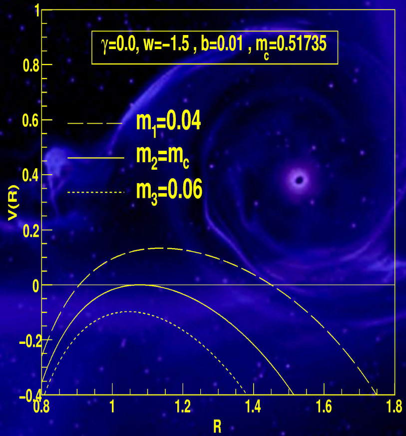


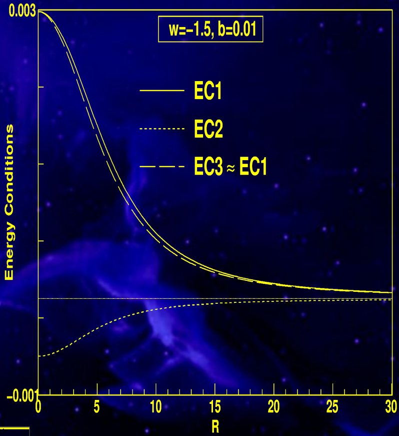


Case G: The potential $V(R)$ and the energy conditions EC1$\equiv \rho + p_r + 2p_t$, EC2$\equiv \rho + p_r$ and EC3$\equiv \rho + p_t$

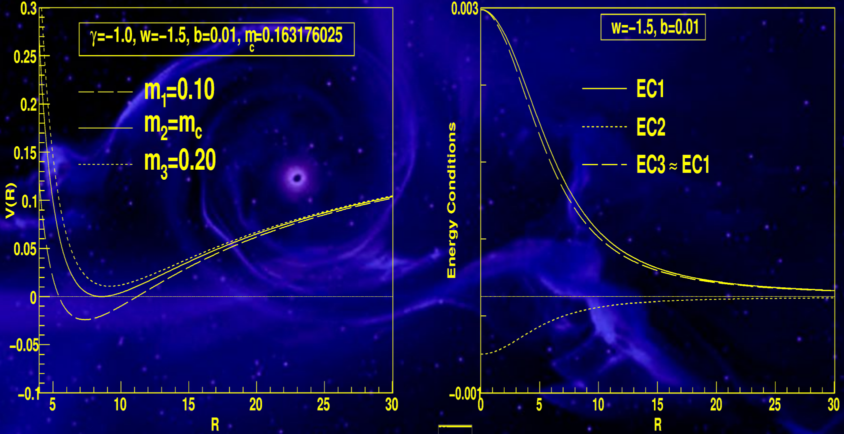
Black Hole or Phantom Gravastar

In this case the potential takes the shape given by figure bellow, from which we can see that $V(R) = 0$ now has four real roots, say, R_i, where $R_{i+1} > R_i$. If we choose $R_0 > R_4$, then again the star will not be allowed in this region because the potential is greater than zero. However, if we choose $R_3 < R_0 < R_4$, the collapse will bounce back and forth between $R = R_3$ and $R = R_4$, as in the previous case. But, if we choose $R_2 < R_0 < R_3$, we can note that this region is forbidden because either the potential is imaginary or greater than zero. However, if we choose $R_1 < R_0 < R_2$, the collapse will bounce back and forth between $R = R_1$ and $R = R_2$. If $R_0 < R_1$ the system will collapse until $R = 0$, whereby a Schwarzschild black hole is finally formed.





Case J:The potential $V(R)$ and the energy conditions EC1$\equiv \rho + p_r + 2p_t$, EC2$\equiv \rho + p_r$ and EC3$\equiv \rho + p_t$



Case J:The potential $V(R)$ and the energy conditions EC1$\equiv \rho + p_r + 2p_t$, EC2$\equiv \rho + p_r$ and EC3$\equiv \rho + p_t$

Table: Summary of all possible kind of energy of the interior and of the shell.

Case	Interior Energy	Shell Energy	Figures	Structures
A	Standard	Standard	10	Normal Star
B	Standard	Dark	4	BH/Dispersion
C	Standard	R. Phantom	5	BH/Dispersion
D	Dark	Standard		Interior not found
E	Dark	Dark		Interior not found
F	Dark	R. Phantom		Interior not found
G	R. Phantom	Standard	11	Gravastar
H	R. Phantom	Dark	6	BH/Dispersion
I	R. Phantom	R. Phantom	7	BH/Dispersion
J	A. Phantom	Standard	12	Gravastar or BH
K	A. Phantom	Dark	8	BH/Dispersion
L	A. Phantom	R. Phantom	9	BH/Dispersion

Conclusions

- We have studied the problem of the stability of gravastars by constructing dynamical three-layer models of VW, which consists of an internal phantom fluid, a dynamical infinitely thin shell of perfect fluid with the equation of state $p = (1 - \gamma)\sigma$, and an external Schwarzschild space.

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- We have shown explicitly that the final output can be a black hole, a "bounded excursion" stable gravastar, a Minkowski, or a phantom spacetime, depending on the total mass m of the system, the parameter ω, the constant b, the parameter γ and the initial position R_0 of the dynamical shell.

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- All these possibilities have non-zero measurements in the phase space of m, b, ω, γ and R_0, although the region of gravastars is very small in comparison with that of black holes.

It is interesting to note that we can have black hole formation even with an interior phantom energy for any given γ.

The results obtained here further confirm our previous conclusion:

Even though the existence of gravastars cannot be completely excluded in these dynamical models, our results do indicate that, even if gravastars indeed exist, they do not exclude the existence of black holes.