Codimension 2 braneworld cosmology.

Why modify GR?

Lovelock theory ⊕ Braneworld.

Codimension 2 braneworlds of maximal symmetry

Braneworld cosmology and the modified FRW equation

Conclusions & Open problems.

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Based on CC & Papazoglou
and CC, Kofinas & Papazoglou. (to appear)

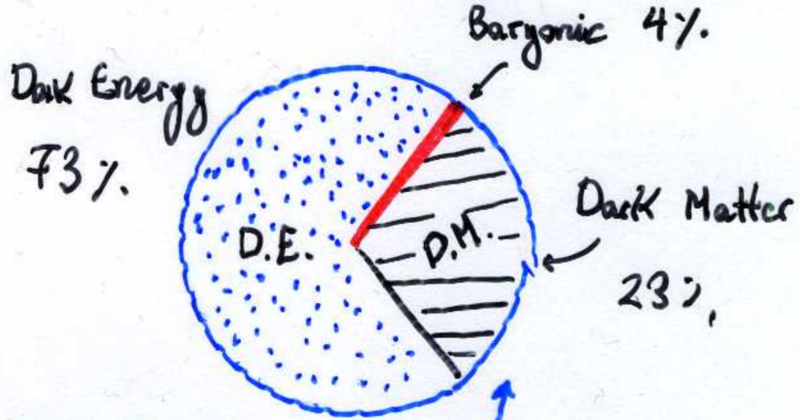
§1 Introduction

Assume GR: $\boxed{R_{ab} - \frac{1}{2} g_{ab} R = 8\pi G T_{ab}}$

Experimental Status $\}$ Excellent agreement very precise tests.

Question: What is the matter content of the Universe today?

Cosmological Observations



Astrophysical data

Answer: Only a mere 4% of matter discovered in the Laboratory

Fact: • If we do not assume extra matter sectors there is tension between cosmological, astrophysical and solar system data

• Universe is accelerating!

Fact: Cosmological data best fitted if we assume a tiny cosmological constant

$$|\Lambda|_{obs} \lesssim (10^{-3}\,eV)^4$$

Part of the cosmological constant problem.

Why such a discrepancy between Λ_{vac} and Λ_{obs}

Why is Λ so small?

Why do we observe it now?

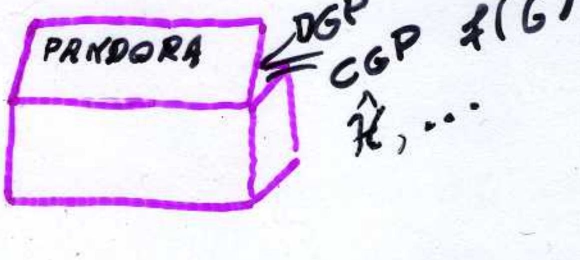
Should we consider exotic matter? dark energy.

Should we explain Λ_{obs} independently of the CC problem?

Can it be that gravity is modified at large scales?

$$\frac{\text{Solar System Scales} \sim 1\ \text{A.U.}}{\text{Cosmological Scales} \sim H_0^{-1}} = (1\ \text{A.U.}\ H_0^{-1}) \sim 10^{-15}$$

Experimentally it is possible, however, numerous consistency problems



Lovelock theory ⊕ Brane world

Two main ingredients : Extra dimensions
Modification of E-H action + distributional sources.

Q What is Lovelock theory?

- It is exactly $GR + \Lambda$ in $D=4$
- It is the generalisation of GR in $D>4$

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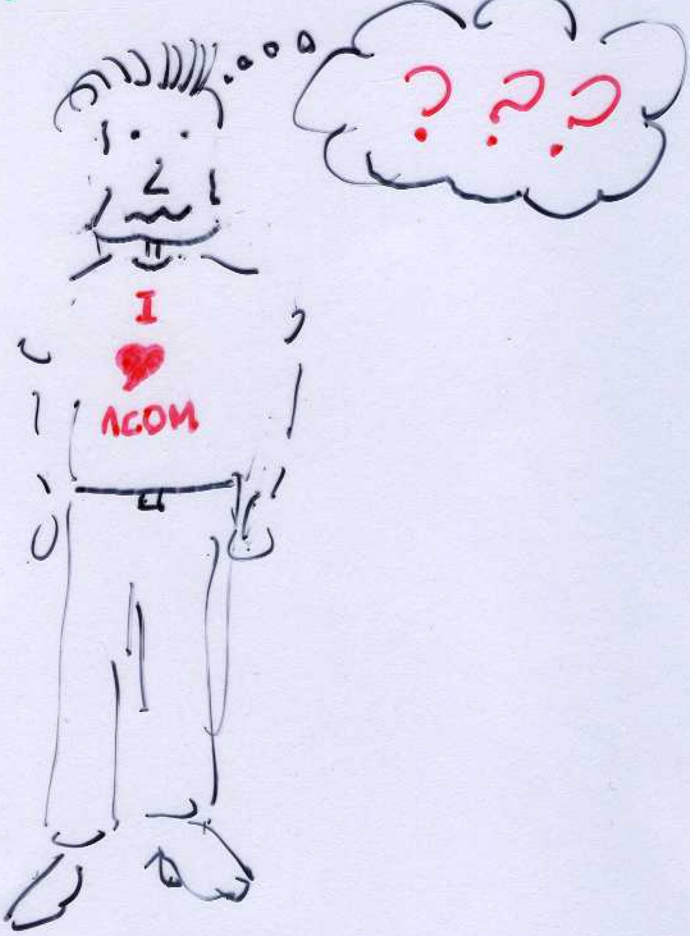


Lovelock theorem:

Given $\mathcal{L}(M, g, \nabla)$ in $D=4$ the Einstein Hilbert action

$$\int_M d^4x \sqrt{-g}\,(-2\Lambda + R) \longrightarrow G_{\mu\nu} + \Lambda g_{\mu\nu}$$

yields the unique metric tensor which is

- symmetric
- divergence free
- depends up to second order derivatives in $g_{\mu\nu}$

Lovelock theorem: (in $D=5,6$]

Given $\mathcal{L}(M, g, \nabla)$ in $D=5, 6$ the EGB action

$$\int d^6x \sqrt{-g} \, (-2\Lambda + R + \alpha \hat{G})$$

yields the unique metric tensor which is

- symmetric
- divergence free
- depends up to second order derivatives in $g_{\mu\nu}$

with $\hat{G} = R^2 - 4R_{\alpha\beta}R^{\alpha\beta} + R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}$

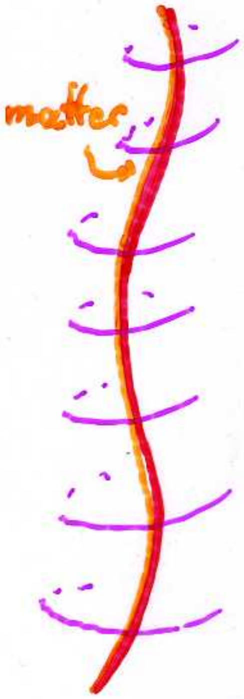
$$\int_{M_2} R \sim \chi(M_2)$$

, ...

$\chi = 2-2h$ $\quad \partial M_2 = \emptyset$

$$\int_{M_4} \hat{G} \sim \chi(M_4)$$ etc.

Codimension 2 braneworlds.



A 4 dimensional braneworld embedded in a 6 dimensional manifold

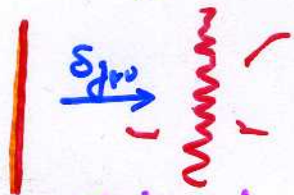
Geometrically similar to vortices in condensed matter or cosmic strings in cosmology.

Q: What are the gravitating braneworlds of maximal symmetry?

Perturbation theory

Exact Solutions

Brane Cosmology



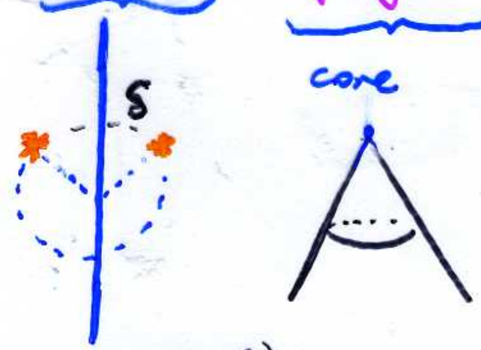
gravitational spectrum and stability

Codimension 2 braneworlds

Analogy with cosmic strings: $ds^2 = \underbrace{-dt^2 + dz^2}_{} + \underbrace{d\rho^2 + \rho^2\beta^2 d\theta^2}_{\text{core}}$

(Vilenkin)

Tension $8\pi G_4 T = -2\pi(1-\beta)$ Conical deficit



$\delta = 2\pi(1-\beta)$

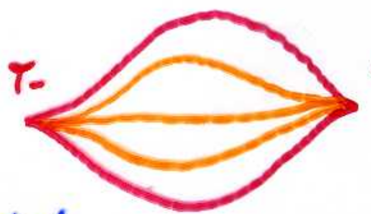
Here for $D=6$ we have a warped cod 2 braneworld.

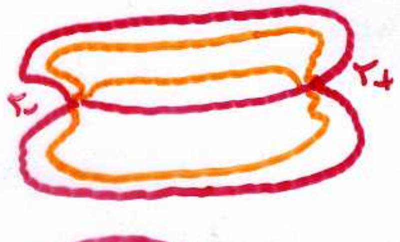
$$ds^2 = \underbrace{r^2}_{\text{Warp factor}} \underbrace{h_{\mu\nu}dx^\mu dx^\nu}_{\text{Brane}} + \underbrace{V(r)\,d\theta^2 + \frac{dr^2}{V(r)}}_{\text{cone?}}$$

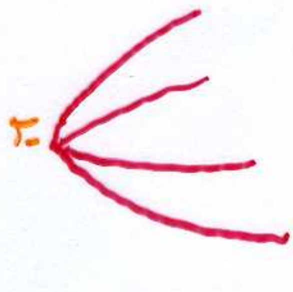
- Brane Position at $r = r_h$ where $V(r_h) = 0$.

Set $\rho = \sqrt{\dfrac{2(r-r_h)}{V'(r_h)}}$ then $ds_2^2 \simeq \frac{1}{4}V'^2(r_h)\,\rho^2 d\theta^2 + d\rho^2$

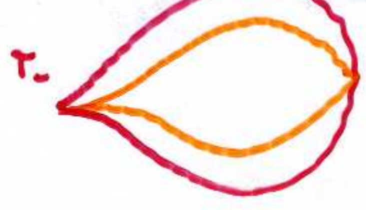
Hence $\beta = \frac{1}{2}|V'(r_h)|$







Carrol et al.
Cline et al
Navarro et al.



see Mukohyama et al. for GR version.

Bostock, Gregory, Navarro, Santiago
CC, Zeyers

Junction conditions

$$2\pi(1-\beta)\left[-S_\mu^\nu \right] = 8\pi G_6 T_\mu^\nu$$

localised matter source

Bostock, Gregory, Navarro, Santiago

CC, Zegers

Junction conditions

$$2\pi(1-\beta)\Big[\underbrace{-\delta_\mu^\nu}_{\text{Einstein part}} + \frac{2\alpha}{3}\underbrace{(G_\mu^\nu + W_\mu^\nu)}_{\hat{G}\text{ part.}}\Big] = 8\pi G_6 \underbrace{T_\mu^\nu}_{\text{localised matter source}}$$

$$W_\mu^\nu = K_\mu^\lambda K_\lambda^\nu - K K_\mu^\nu + \frac{1}{2}\delta_\mu^\nu (K^2 - K_{\alpha\lambda}^2)$$

$$S_{bulk} = \int d^4x\, d^2y\, \sqrt{-g}\,\big[-2\Lambda + R + \alpha\,\hat{G}\big] + \int d^4x\, \mathcal{L}_{matter}$$

distributional part

cod 2 cod 2

$$2\pi(1-\beta)\,\delta^2(y)\int d^4x\,\sqrt{-h}\,\big[\,1 + \alpha(R_{ind} + W_{ind})\big]$$

tension E-H action + ..

Distributional part of bulk Lovelock densities are induced lower order Lovelock densities.

Example : de Sitter brane.

J.C. $\quad 2\pi(1-\beta)\left[-\delta_\mu^\nu + \frac{2\alpha}{3} G_\mu^\nu{}^{(ind)}\right] = 8\pi G_6 T_\mu^\nu$

$$G_{\mu\nu}^{(ind)} = -3H_0^2 h_{\mu\nu}^{(4)} = -\frac{3}{r_h^2} h_{\mu\nu}^{(4)}$$

$$ds^2 = V(r)\,d\theta^2 + \frac{dr^2}{V(r)} + r^2 h_{\mu\nu}\,dx^\mu dx^\nu, \quad V(r_h) = 0.$$

Where $r_h = r_h(\alpha, \Lambda, \mu)$ is given by the soliton solution

For $\alpha \neq 0$ $\quad G_4 = \frac{3G_6}{4\pi\alpha(1-\beta)}$ 4 dim Newton constant

(J. Cond.) $\quad T = \frac{3}{8\pi G_4}\left[\frac{1}{2\alpha} + H_0^2\right]$

Vacuum Energy

Geometric Contribution.

Effective 4 dim expansion

What about cosmology?

Codimension 2 brane cosmology:

We have hints that : - standard FRW may be recovered
- geometric self-acceleration is possible

There are hints not facts and we need some handle on the cosmology.

because :

For codimension 2 junction conditions provide conditions but not the full information in order to establish the brane dynamics.

Idea is to analyse the coupled system of field equations and junction conditions close to the brane

$$S = \frac{1}{2\kappa_6^2} \int d^6x \sqrt{-g} \left(R - 2\Lambda_6 + \frac{\alpha}{6} \hat{G} \right) + \delta^2 \int_4 \mathcal{L}^{(4)}_{matter}$$

Bulk
pure gravitational
background.

4 dim matter
perfect fluid
$\rho = \rho(t)$
$P = P(t)$

Consider the cosmological metric Ansatz.

$$ds^2 = \underbrace{-n^2(t,r)\,dt^2 + a^2(t,r)\,dx_3^2}_{} + \underbrace{dr^2 + L^2(t,r)\,d\theta^2}_{\text{xtra dimension.}}$$

with
$$n(t,r) = 1 + N(t)\,r + \frac{1}{2} N_2(t)\,r^2 + \mathcal{O}(r^3)$$
$$a(t,r) = a(t) + a(t)\,A(t)\,r + A_2(t)\,r^2 + \mathcal{O}(r^3)$$
$$L(t,r) = \mathcal{B}(t)\,r + \frac{1}{2}\mathcal{B}_2(t)\,r^2 + \mathcal{O}(r^2)$$

Conical geometry with time varying deficit

As the brane evolves so does the deficit angle.
$N(t)$, $A(t)$ are extrinsic curvature terms describing the bending of the brane in the bulk

Input in Lovelock eq$\underline{^{ons}}$ $\mathcal{L}_{\alpha\beta} = \delta^{(2)}_{(r)}\, S^{(4)}_{\alpha\beta}$

$$\hat{\mathcal{L}}_{\alpha\beta} = \hat{\mathcal{L}}_{\alpha\beta}\left(\frac{1}{r}\right) + \hat{\mathcal{L}}_{\alpha\beta}(1) + \mathcal{O}(r)$$

where $S^{(4)}_{\alpha\beta} = \begin{cases} S^{(4)}_{\mu\nu} & \text{perfect fluid in } (t, \vec{x}_3) \\ 0 & \text{otherwise} \end{cases}$

$$\mathcal{L}_{\alpha\beta} = d[\mathcal{L}_{\alpha\beta}]\,\delta^{(2)}(r) + \hat{\mathcal{L}}_{\alpha\beta}$$

For $r \to 0$ $\begin{cases} dist\,[\mathcal{L}_{\alpha\beta}] = S_{\alpha\beta} \\ \mathcal{L}_{\alpha\beta}\left(\frac{1}{r}\right) = 0 \\ \mathcal{L}_{\alpha\beta}(1) = 0. \end{cases}$

→ the system closes and is mathematically consistent!

$$\frac{\kappa_6^2(\rho+\lambda)}{3\alpha(1-\beta)} = H^2 + \frac{k}{a^2} + \frac{1}{2\alpha} - A^2$$

$$-\frac{\kappa_6^2(\rho+3\bar{P}-2\lambda)}{6\alpha(1-\beta)} = \frac{\ddot{a}}{a} + \frac{1}{2\alpha} - f A^2$$

Junction Condition

Bulk constraint

$$2\dot{A}A + 2H(1-f)A^2 = \frac{\dot{\beta}}{\beta}\left(H^2 + \frac{1}{2\alpha} - A^2\right)$$

$$N = fA \quad , \quad f = \frac{3\bar{P}-3\lambda}{\rho+\lambda}.$$

$$\dot{\rho} + 3H(\rho+\bar{P}) + \frac{\dot{\beta}}{\beta(1-\beta)}\rho = 0.$$

Energy (non) conservation

λ is tension

α $\hat{G}$ coupling.

$k = 0, 1, -1$

$a = a(t)$

$\rho = \rho(t)$, $\bar{P} = \bar{P}(t)$

$N = N(t)$, $A = A(t)$

$\beta = \beta(t)$

N_2 , A_2 , β_2 uniquely given by next order

The system is consistent with an arbitrary bulk function $\beta = \beta(t)$!

$$G_4 = \frac{\kappa_6^2}{\alpha(1-\beta)}$$

↳ time varying G_4.

Consider $\dot{\beta}=0$. as a precise example. Solution is

$$\boxed{H^2+\frac{\kappa}{a^2}=-\frac{1}{2\alpha}+\cancel{\frac{\lambda\kappa_6^2}{3\kappa(1-\beta)}}^{,0}+\rho(t)\frac{\kappa_6^2}{3\alpha(1-\beta)}+\frac{c^2}{a^{2(1-3w)}}}$$

$$\boxed{\frac{\ddot{a}}{a}=-\frac{1}{2\alpha}+3w\frac{c^2}{a^{2(1-3w)}}-\rho\frac{(1+3w)}{6\alpha(1-\beta)}\kappa_6^2+\cancel{\frac{\lambda\kappa_6^2}{3\alpha(1-\beta)}}^{,0}}$$

w	$\rho(t)$	A^2
$w=-1$	constant	$\frac{c^2}{a^8}$
$w=\frac{1}{3}$	radiation a^{-4}	c^2 constant
$w=0$	a^{-3} matter	c^2/a^2 curvature or strings
$w=-\frac{1}{3}$	a^{-2} strings or curvature	$\frac{c^2}{a^4}$ radiation
$w=-\frac{2}{3}$	a^{-1} domain wall	$\frac{c^2}{a^6}$

Ordinary perfect fluid yields a two-component perfect fluid!

Conclusions.

- Cod 2 braneworld cosmology mathematically consistent
- Higher oder Lovelock terms yield Einstein like gravity for codimension 2 braneworlds.
- Geometric self-acceleration, is possible at the price of a strong hierarchy between the Einstein and $\hat{G}$ term.
- Junction conditions of Bostock et al introduce cod 1 distributional terms.
- Exact solutions necessary to understand bulk geometry
- Modified LFRW eq$^{\underline{ans}}$

  $\dot{\sigma} \neq 0$ non conservation of energy on the brane

  2 component perfect fluid.

  geometric acceleration.