Cosmic Superstrings: Dynamics and cusps

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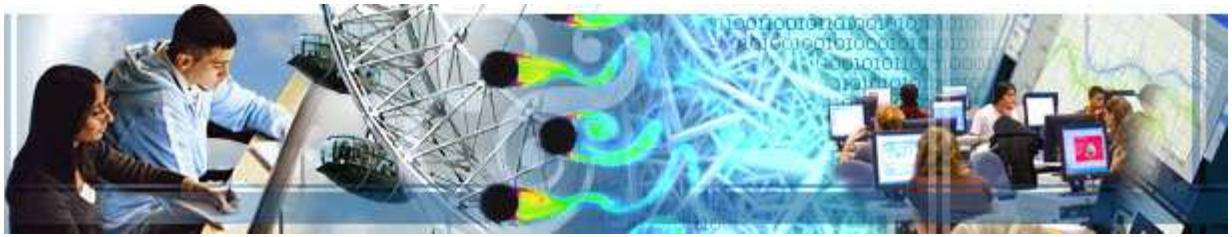

Based on the paper:

A. C. Davis[1], W. N., S. Rajamanoharan[1] and M. Sakellariadou[2],

'*Cusps on cosmic superstrings with junctions,*'

JCAP **0811** (2008) 022 [arXiv:0809.2263 [hep-th]].

[1] DAMTP, University of Cambridge, [2] King's College London.

Standard Cosmic Strings

- Cosmic strings are a type of topological defect that form in the early universe.
- They form in exactly the same way as grain boundaries in crystals.
- As a simple example of how they form, consider (the easy, well understood problem of) water

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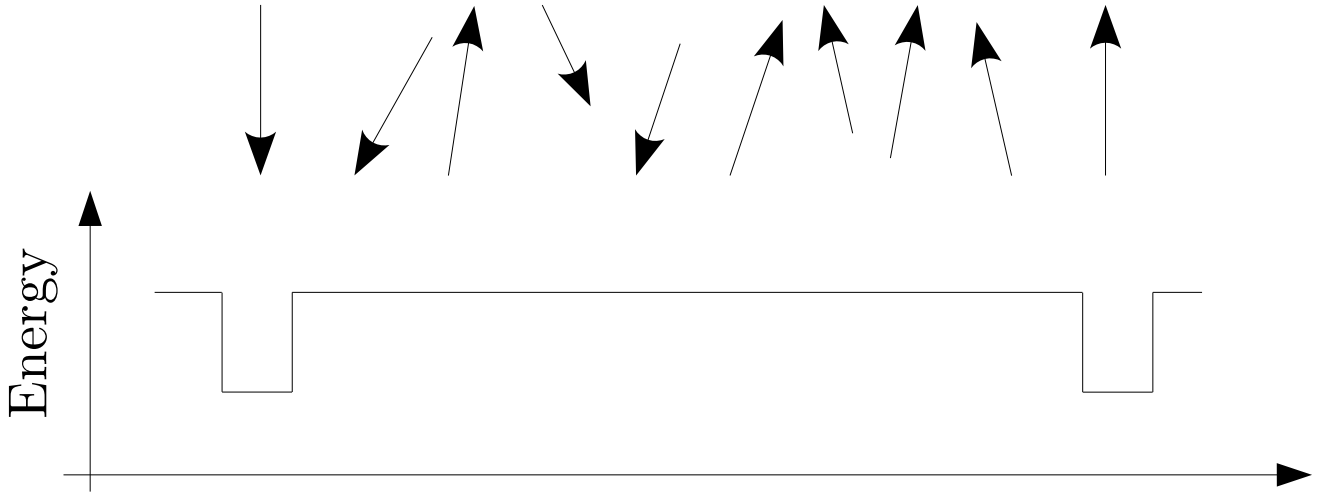
Simple model: phase transition

Above a certain temperature all possible directions of the water molecule have equal energy (symmetric phase), but below the freezing temperature the crystal prefers particular directions (the symmetry is broken), then a topological defect can grow.



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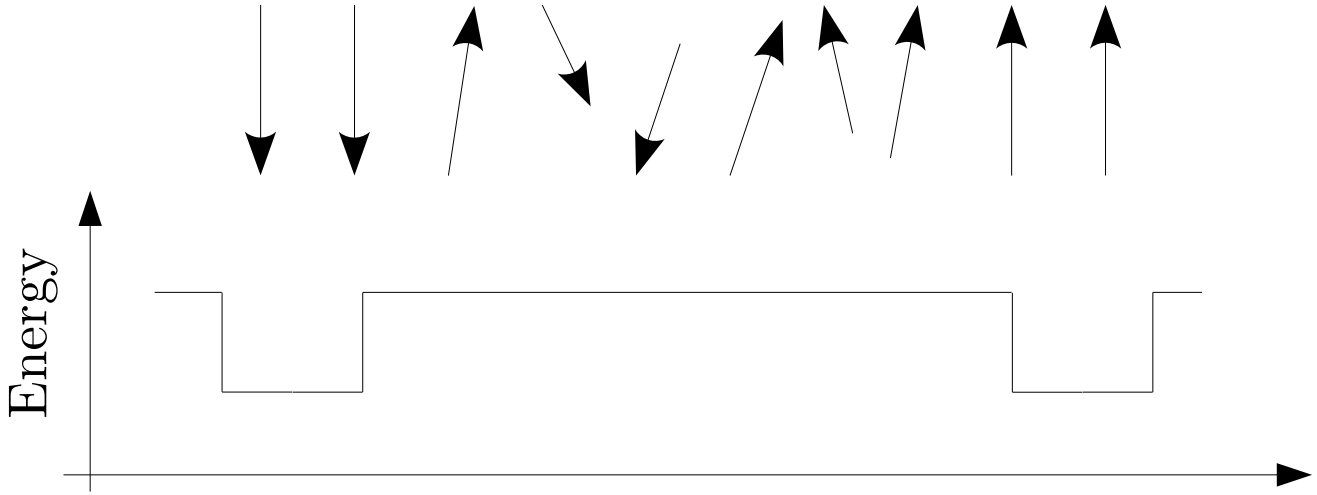

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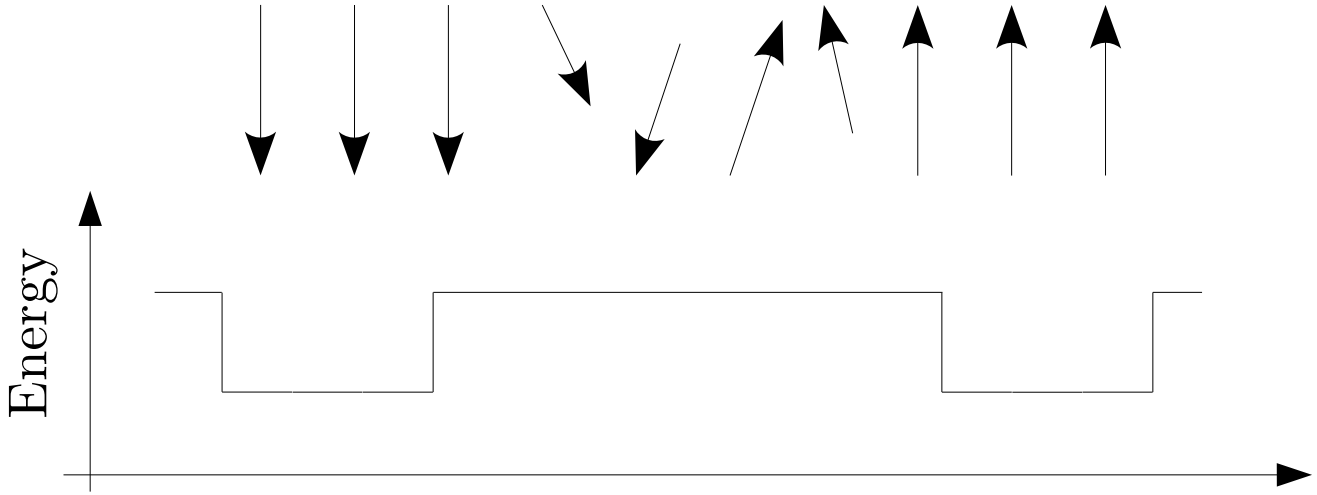

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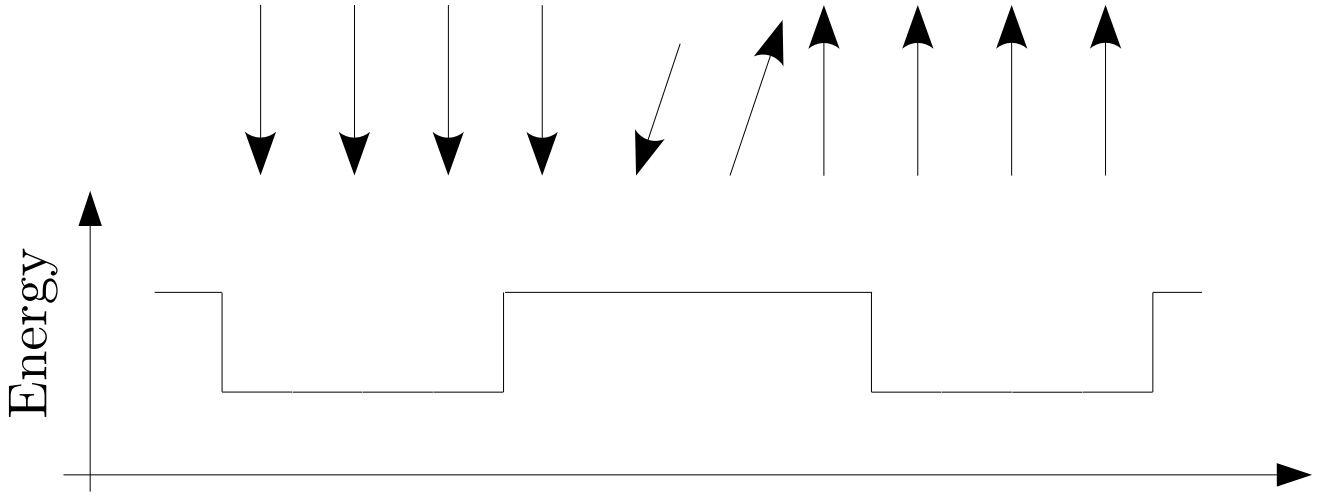

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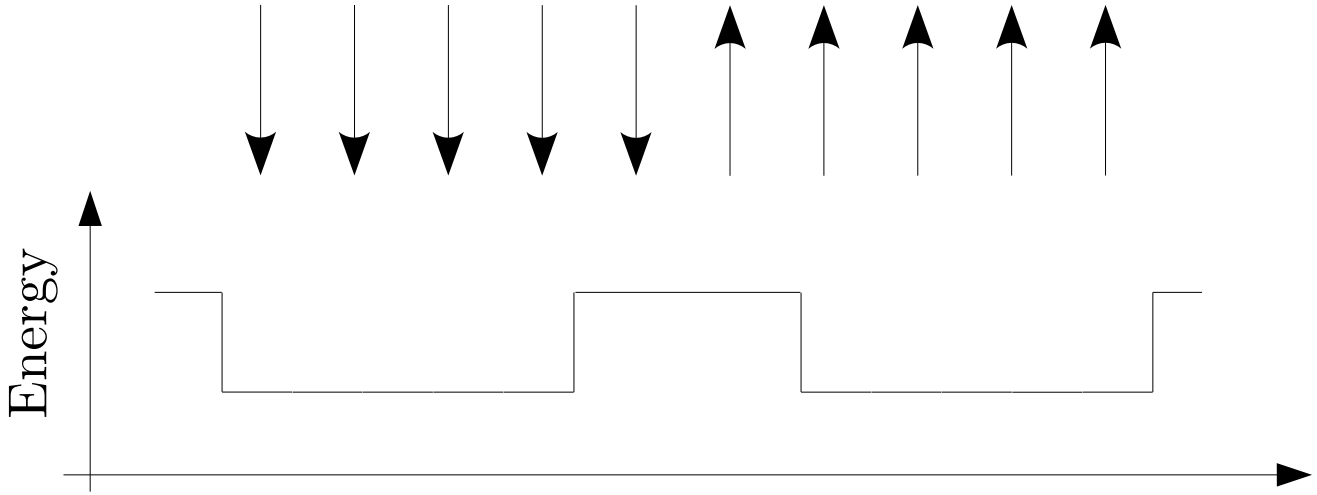

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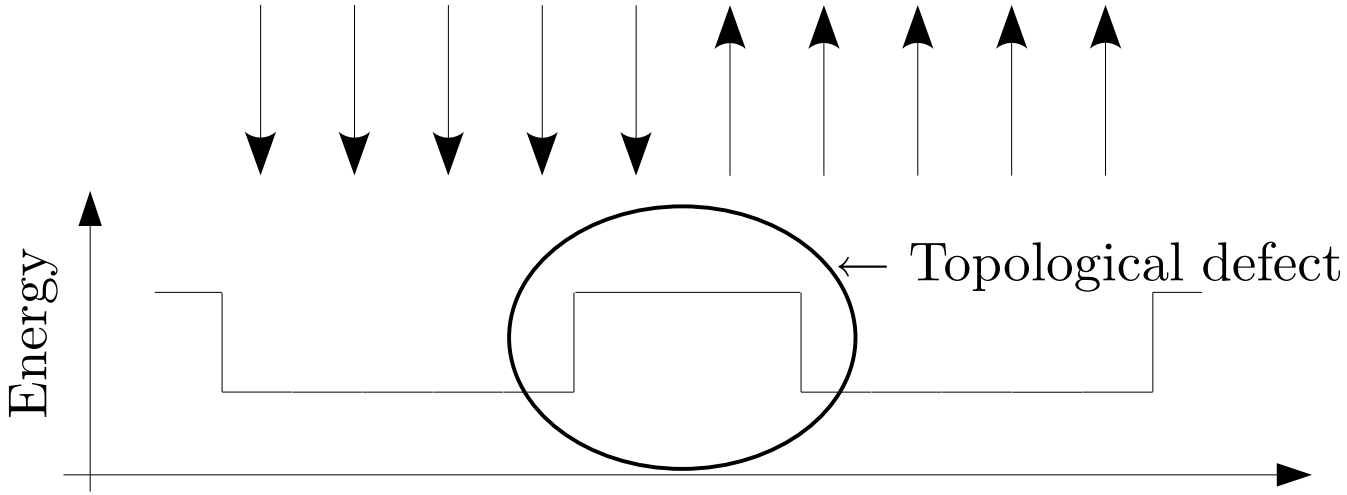
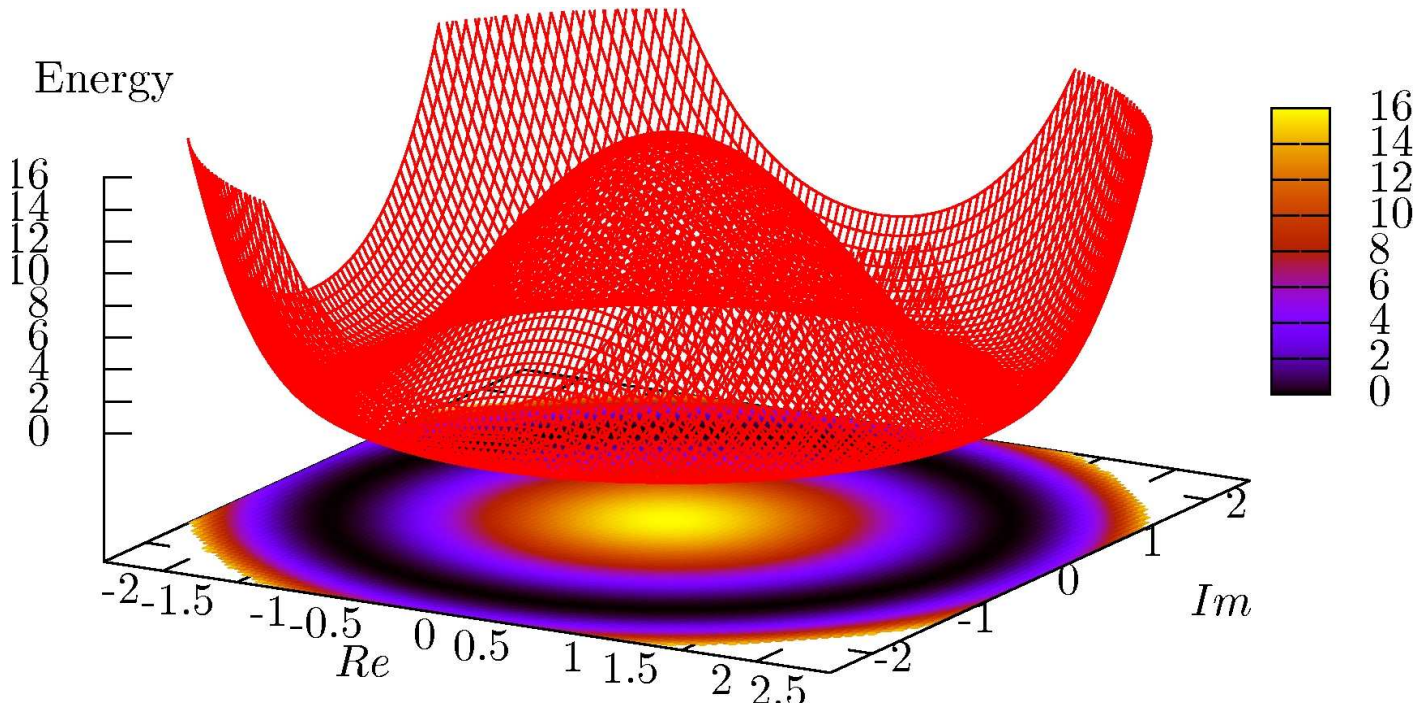


Topological Defects

For a complex field, every point has a magnitude and phase i.e. every point is a 2 dim. vector.

So in this case we can have one dimensional defects: strings.

For them to form, we need the energy of the broken symmetry phase to be zero when the magnitude of the field is non-zero.



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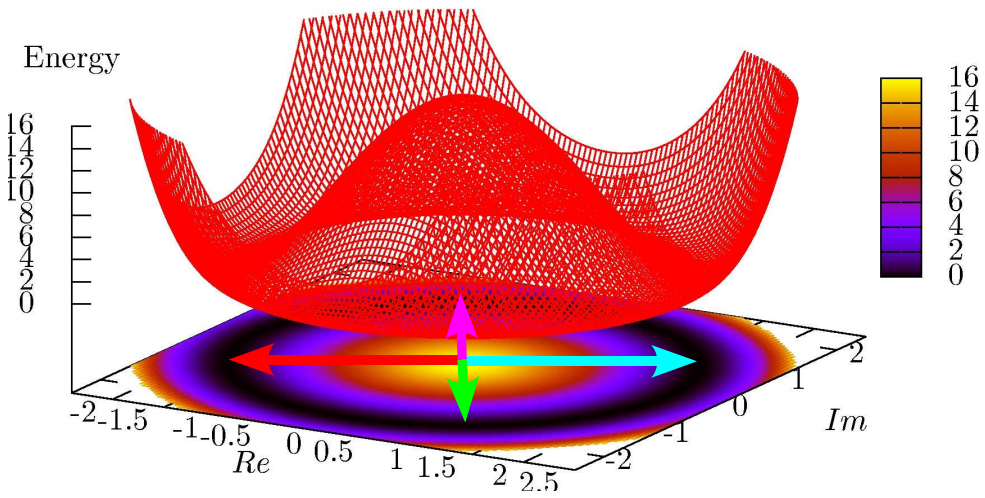

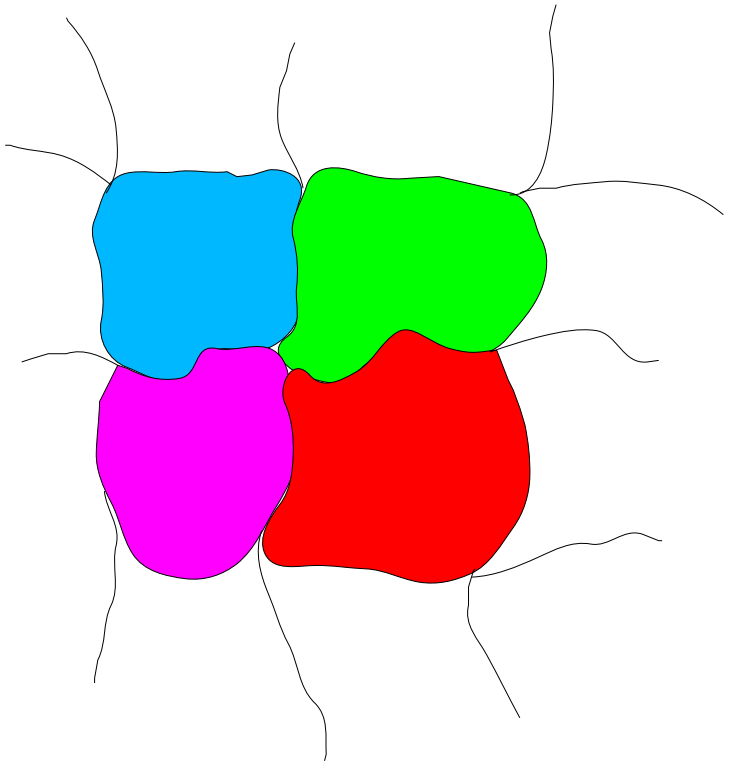

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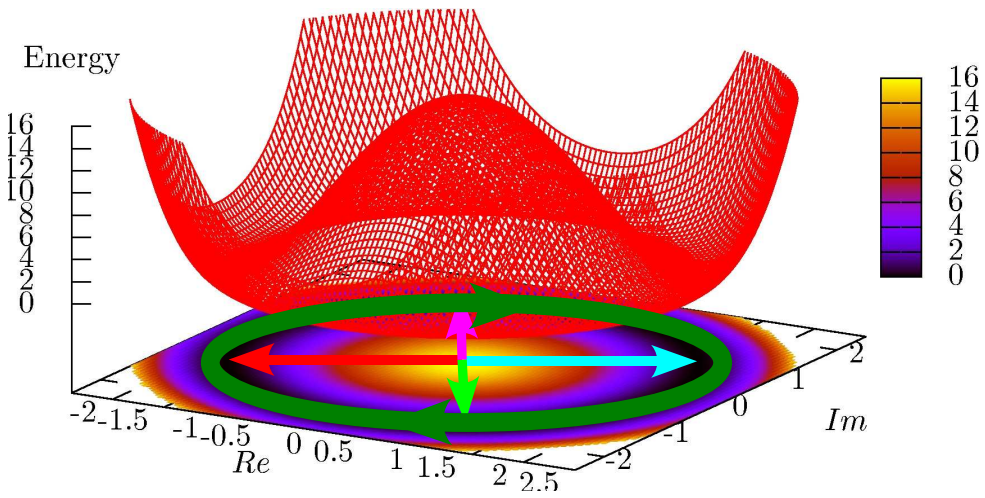

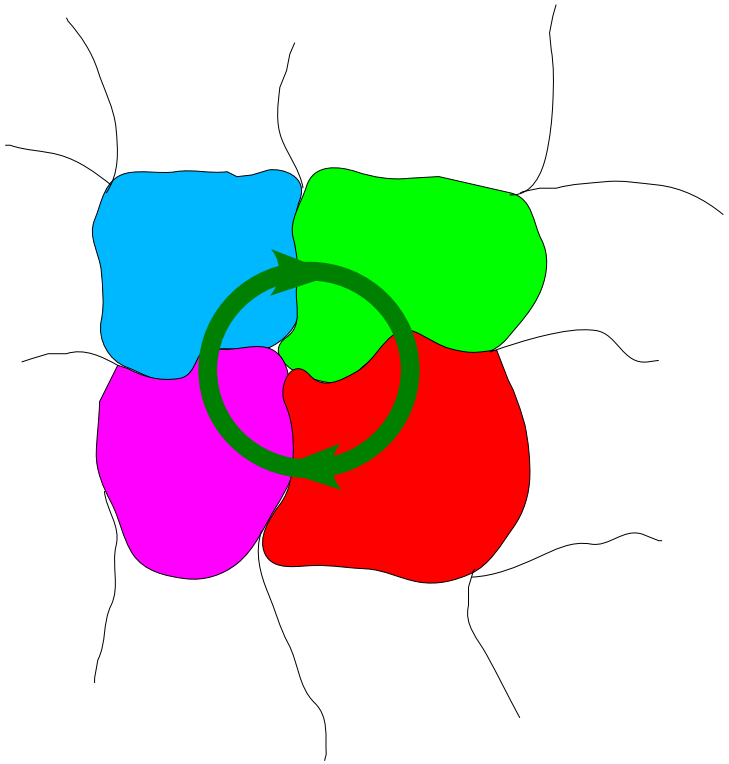

Topological Defects

- The phase must be uncorrelated over distances larger than the horizon
- For a non-zero winding, continunity requires that a topological defect forms
- For a complex field, this defect is a Cosmic String
- For most GUT $\rightarrow$ standard model symmetry breaking schemes, Cosmic Strings form

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Standard Cosmic Strings

Cosmic Strings are (almost) one dimensional topological defects. Neglecting their thickness their dynamics are given by the Nambu-Goto action,

$$S = \mu \int \mathrm{d}\sigma \mathrm{d}\tau \sqrt{-\gamma} \, ,$$

where σ is a spacelike and τ a timelike coordinate on the world sheet and $\gamma_{\alpha\beta}$ is the world sheet metric.

Standard Cosmic Strings

Gauge fixing: Minkowski

We can fix the gauge freedom by setting:

$$\dot{x} \cdot x' = 0 \,, \qquad \dot{x}^2 + x'^2 = 0 \,. \quad \text{(Conformal gauge)}$$

And $\tau = t = x^0$ (Static gauge).

In this gauge the equations of motion become the standard wave equation,

$$\ddot{\mathbf{x}} - \mathbf{x}'' = 1 \,,$$

where $x(t, \sigma) = (t, \mathbf{x}(t, \sigma))$.

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Exactly as in a violin string, this is solved for,

$$\mathbf{x}(t,\sigma) = \frac{1}{2}\left(\mathbf{a}(t-\sigma) + \mathbf{b}(t+\sigma)\right) \ ,$$

where **a** and **b** are arbitrary functions (left and right movers).
Using this notation the gauge constraints become,

$$\left|\mathbf{a}'\right|^2 = \left|\mathbf{b}'\right|^2 = 1 \ ,$$

where here the $'$ is the total derivative.
So the vectors $\mathbf{a}'$ and $\mathbf{b}'$ are unit vectors.

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Standard Cosmic Strings: Loops

Just as for a violin string we need to specify boundary conditions to solve the system.

Boundary conditions

Infinite strings	Loops
$-\infty < \sigma < \infty$	$\sigma = \sigma + L$

Where $L = \mathcal{E}/\mu$ is the (coordinate) length of the string.
(For expanding spacetimes $\mathcal{E}$ depends on $\dot{a}$)

For standard cosmic strings the interesting case is loops.

In terms of **a** and **b** the periodicity requirement is,

$$\mathbf{a}(t-\sigma-L) - \mathbf{a}(t-\sigma) = -\Big(\mathbf{b}(t+\sigma+L) - \mathbf{b}(t+\sigma)\Big) = \text{const}\ ,$$

where the constant is proportional to the momentum of the loop.

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Standard Cosmic Strings

Choosing the centre of mass frame of the loop so that the momentum is zero we find that both **a** and **b** are periodic in $\sigma \to \sigma + L$.

and hence

$$\int_0^L \mathbf{a}' \mathrm{d}\sigma = \int_0^L \mathbf{b}' \mathrm{d}\sigma = 0 \ ,$$

Finally then, $\mathbf{a}'$ and $\mathbf{b}'$ are periodic unit vectors which have a zero 'centre of mass'.
We can visualise them as closed loops on a sphere.

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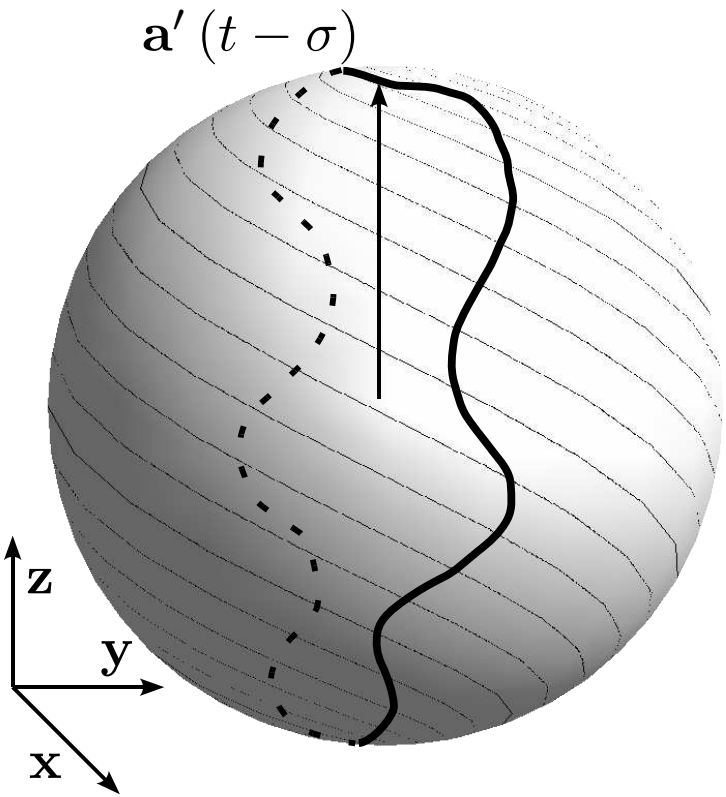

The closed curve specifies $\mathbf{a}'(t-\sigma)$ for all σ at a particular time t.

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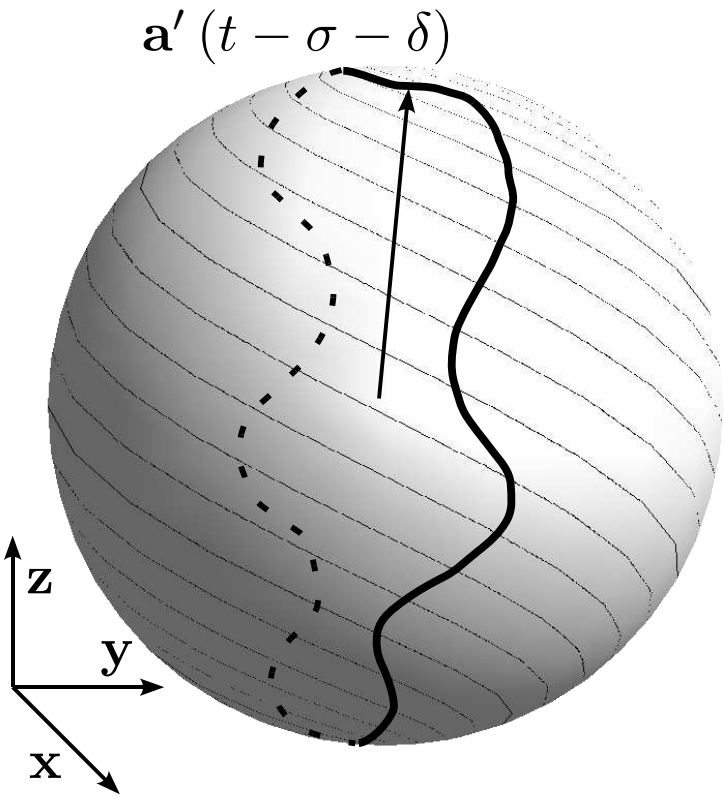

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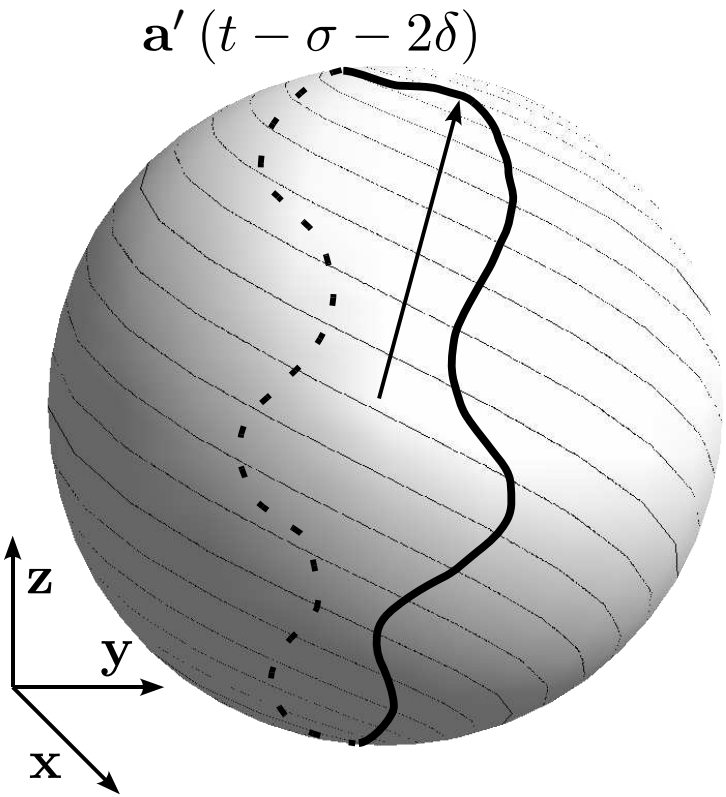

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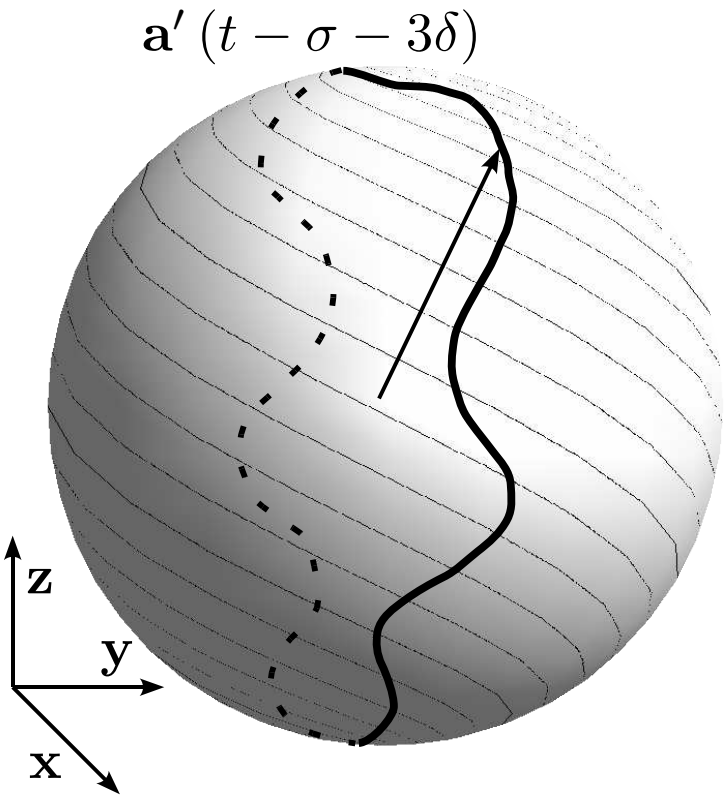

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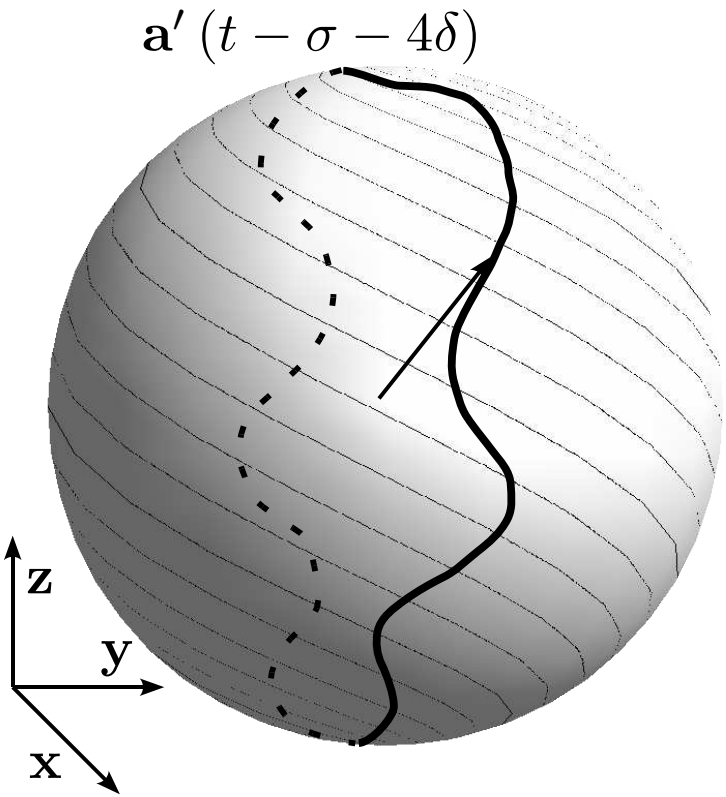

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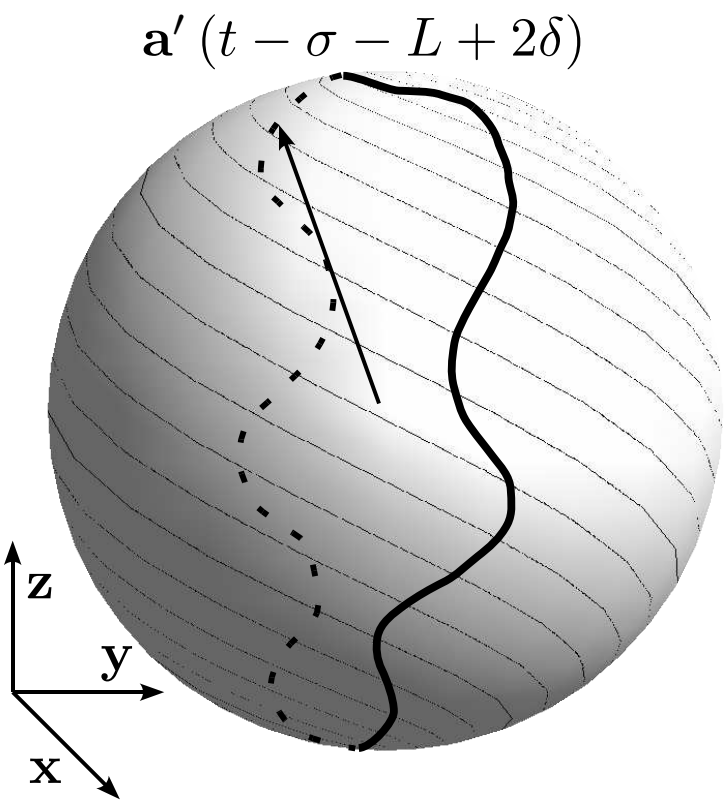

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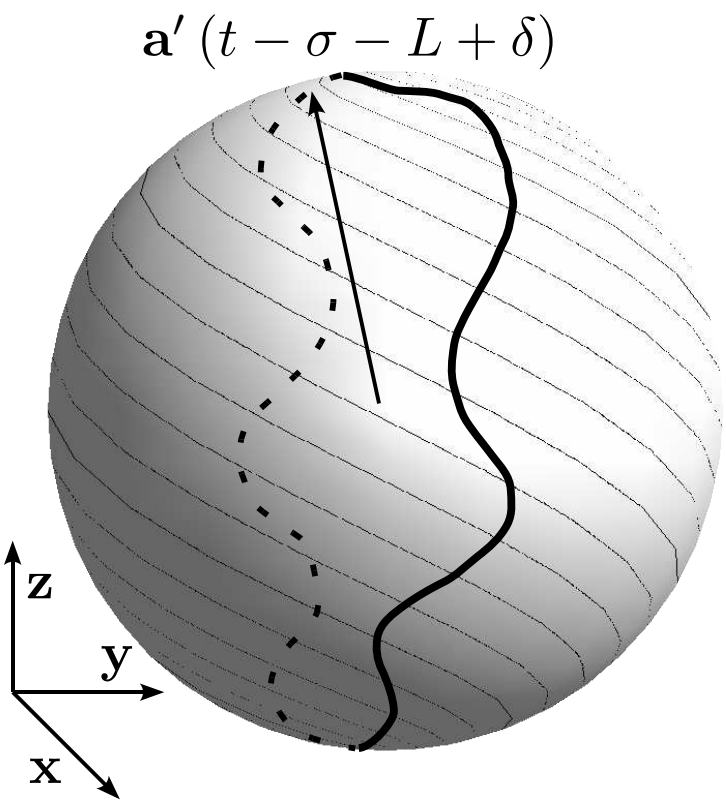

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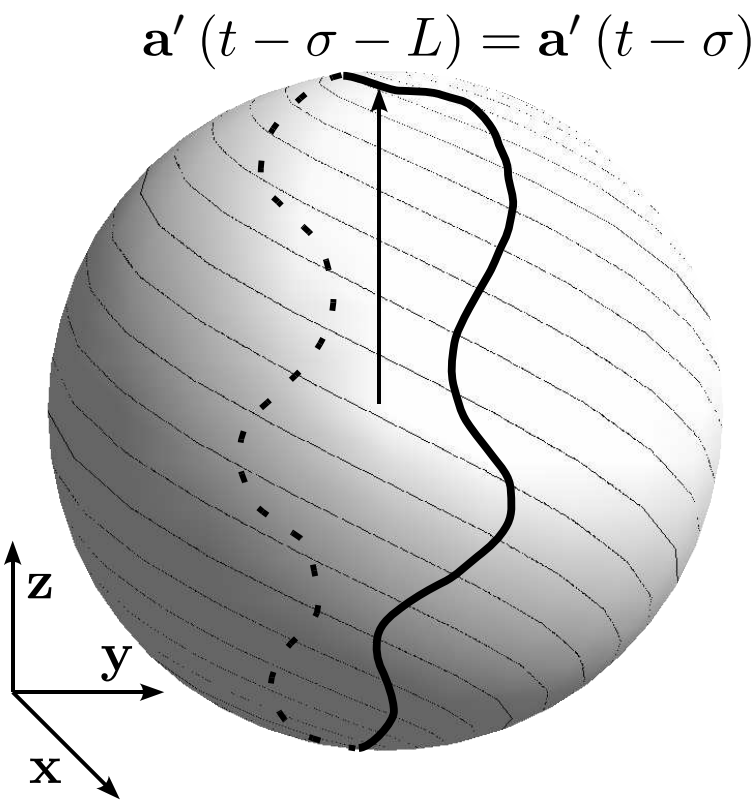

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We will use this type of picture for the more complicated cases later, but first:

Cusps

The velocity of a point is given by,

$$|\dot{\mathbf{x}}| = \frac{1}{2}\left|\mathbf{a}' + \mathbf{b}'\right| ,$$

So if $\mathbf{a}' = \mathbf{b}'$, we get

$$|\dot{\mathbf{x}}| = \left|\mathbf{a}'\right| = \left|\mathbf{b}'\right| = 1 ,$$

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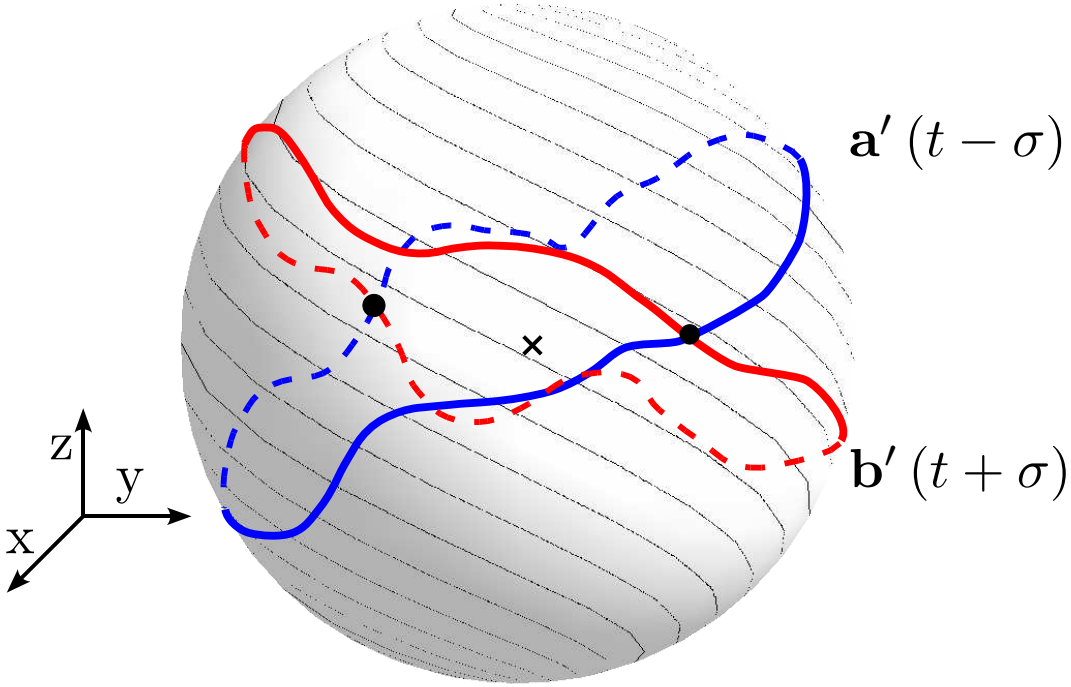

Because $\mathbf{a}'$ and $\mathbf{b}'$ are independent and because the 'centre of mass' of the closed curves are zero,
generically we will get intersections

Cosmic Superstrings

Everything so far is well known, A. Vilenkin, P. Shellard (1994). What we want to look at is what changes for cosmic superstrings.

Cosmic superstrings are either the F-strings of string theory or D-branes that are extended in only one dimension (these could be D1-branes or Dp-branes with $p-1$ dimensions compactified)

In brane inflation models many such defects can occur.

N. Jones, H. Stoica, S. Tye (2002) S. Sarangi, S. Tye (2002)

It is also possible that these defects are embedded within our D3-brane, where they would also look localised.

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Cosmic Superstrings

The defects from the brane inflation models might play the role of cosmic strings. The details of the model are not going to be important.

What is important is that we have two types of cosmic strings and that they carry a conserved electric flux.

This allows the strings to form junctions. But first we'll look at the (low energy) dynamics of single cosmic superstrings.

As cusps emit radiation, they are possible observables and so if we could find cusps on Cosmic Superstrings, we could probe brane inflation models and string theory modes.

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DBI ACTION

The low energy effective action, for a dimenionally reduced string theory, (i.e. a truncation to ignore the massive KK modes)

$$\mathcal{S} = -\mu \int \mathrm{d}\tau \mathrm{d}\sigma \sqrt{|\gamma_{\alpha\beta} + F_{\alpha\beta}|}\ ,$$

where $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$ is the electromagnetic field of the world sheet gauge fields.

Cosmic Superstrings

We can fix the gauge exactly as before (again for Minkowski spacetime),

$$\dot{x} \cdot x' = 0 \,, \qquad \dot{x}^2 + x'^2 = 0 \,. \qquad \text{(Conformal gauge)}$$
$$x^0 \equiv t = \tau \qquad \text{(Static gauge)}$$

So that the equations of motion are the same: $\ddot{\mathbf{x}} - \mathbf{x}'' = 0$,

But now we also have the equation of motion for the gauge fi elds:

$$p \equiv \frac{\delta \mathcal{L}}{\delta F_{\tau\sigma}} = \frac{\mu F_{\tau\sigma}}{-\mathbf{x}'^2 \left(1 + \dot{\mathbf{x}}^2\right) - F_{\tau\sigma}} \,,$$

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Cosmic Superstrings

Because the world-sheet electromagnetic field is non-dynamical, everything is the same as in the standard cosmic string case.

Boundary conditions

In principle the F-string can end on D-branes.

Later we'll look at F-strings ending on junctions, but as a warm up let's consider an F-string ending on a pair of parallel D2-branes.

Cosmic Superstrings

Because the world-sheet electromagnetic field is non-dynamical, everything is the same as in the standard cosmic string case.

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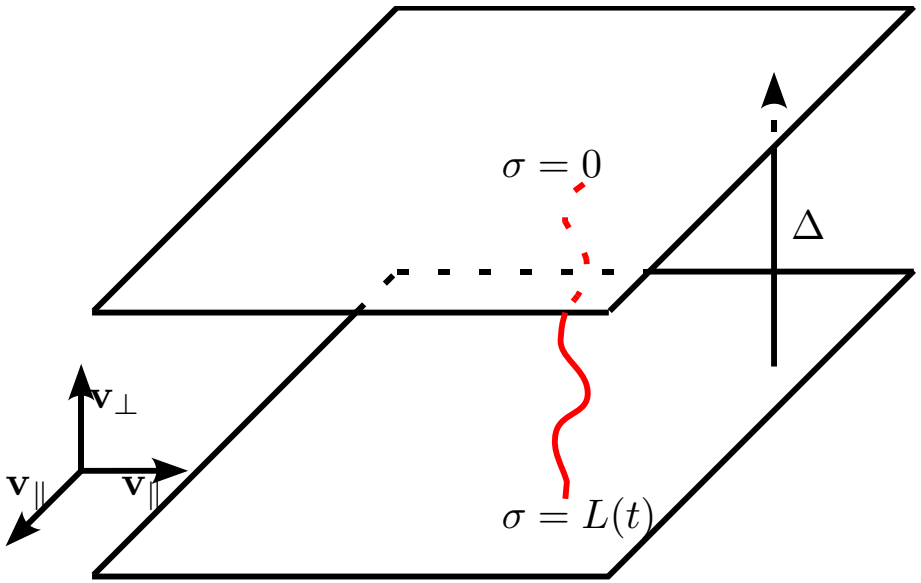
Cosmic Superstrings: Boundary Conditions

The boundary conditions are:

$$\dot{\mathbf{x}}_\perp(t,0) = \dot{\mathbf{x}}_\perp(t,L) = 0,$$
$$\mathbf{x}'_\parallel(t,0) = \mathbf{x}'_\parallel(t,L) = 0\ ,$$

These are Dirichlet and Von Neumann respectively.



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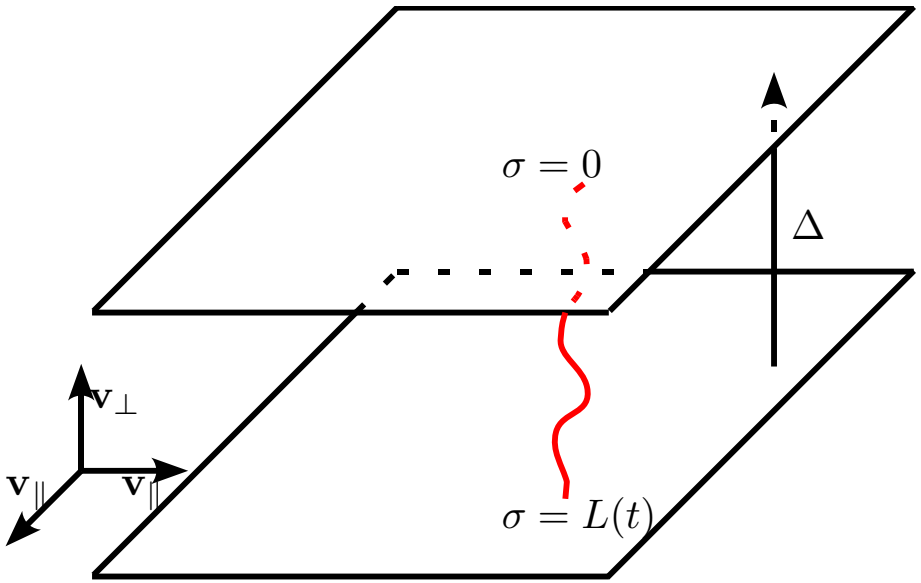

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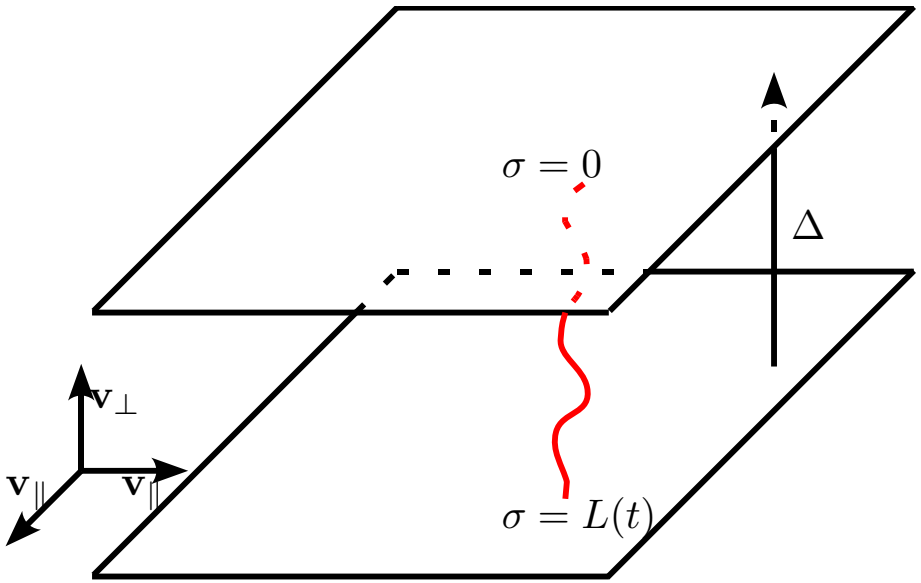

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In terms of $\mathbf{a}'$ and $\mathbf{b}'$ these conditions are:

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Which implies,

$$\mathbf{a}'(t-L) = \mathbf{a}'(t+L) \ , \quad \mathbf{b}'(t-L) = \mathbf{b}'(t+L)$$

So again, $\mathbf{a}'$ and $\mathbf{b}'$ are periodic, but now they are reflections of each other.

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Now we can only choose the centre of mass frame along the directions parallel to the branes.

This changes the averaging conditions we had:

$$\frac{1}{2}\int_{-L}^{L} \mathbf{a}'(\zeta)\,\mathrm{d}\zeta = \frac{\Delta}{L}\,,$$

So the vectors $\mathbf{a}'$ and $\mathbf{b}'$ are closed loops on a unit sphere that are reflections of each other through the brane, and have a 'centre of mass' given by the interbrane separation.

This is true for any dimensionality brane, but we'll look at only D2-branes, which are the simplest.

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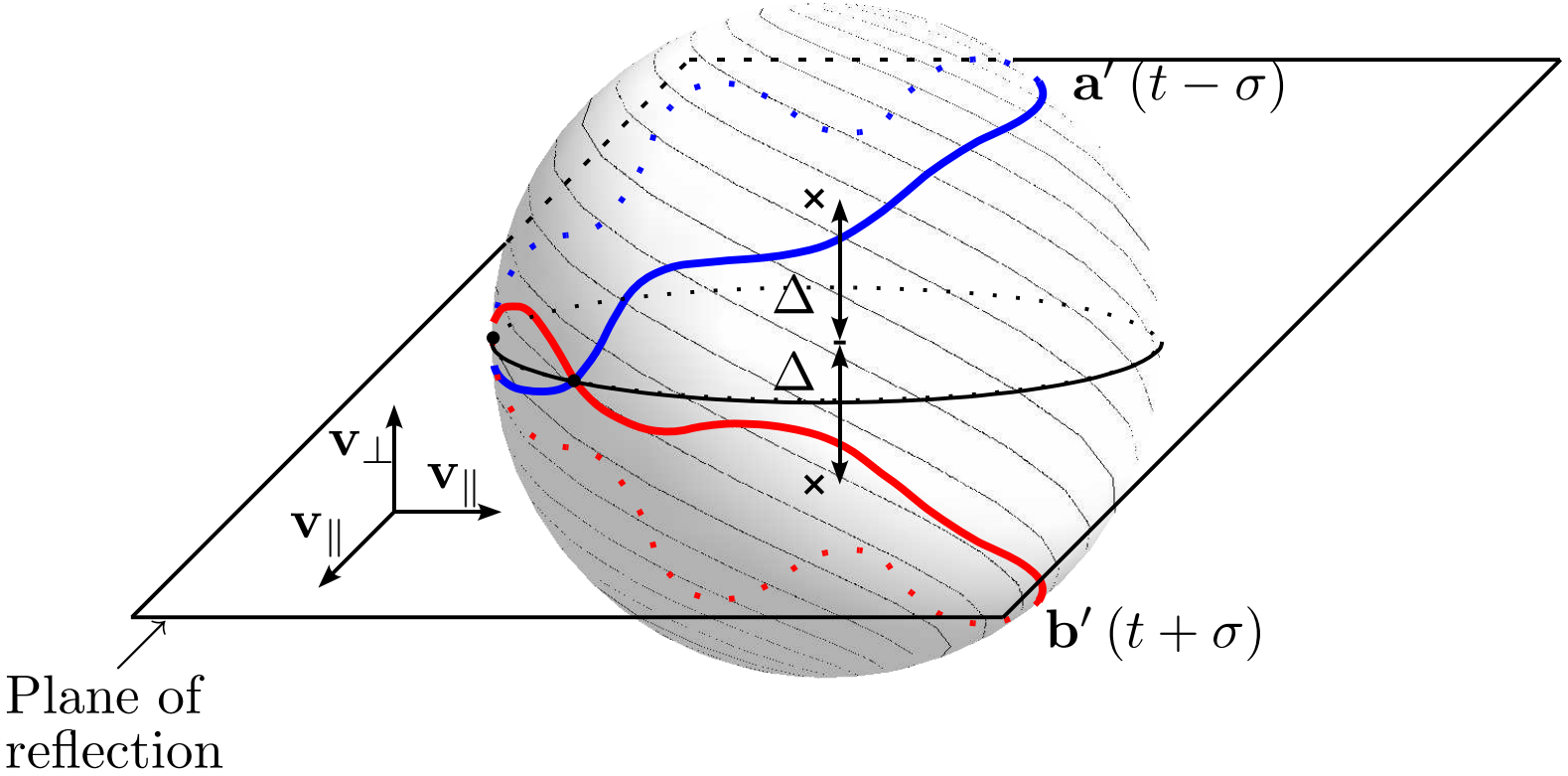
Cosmic Superstrings: Examples



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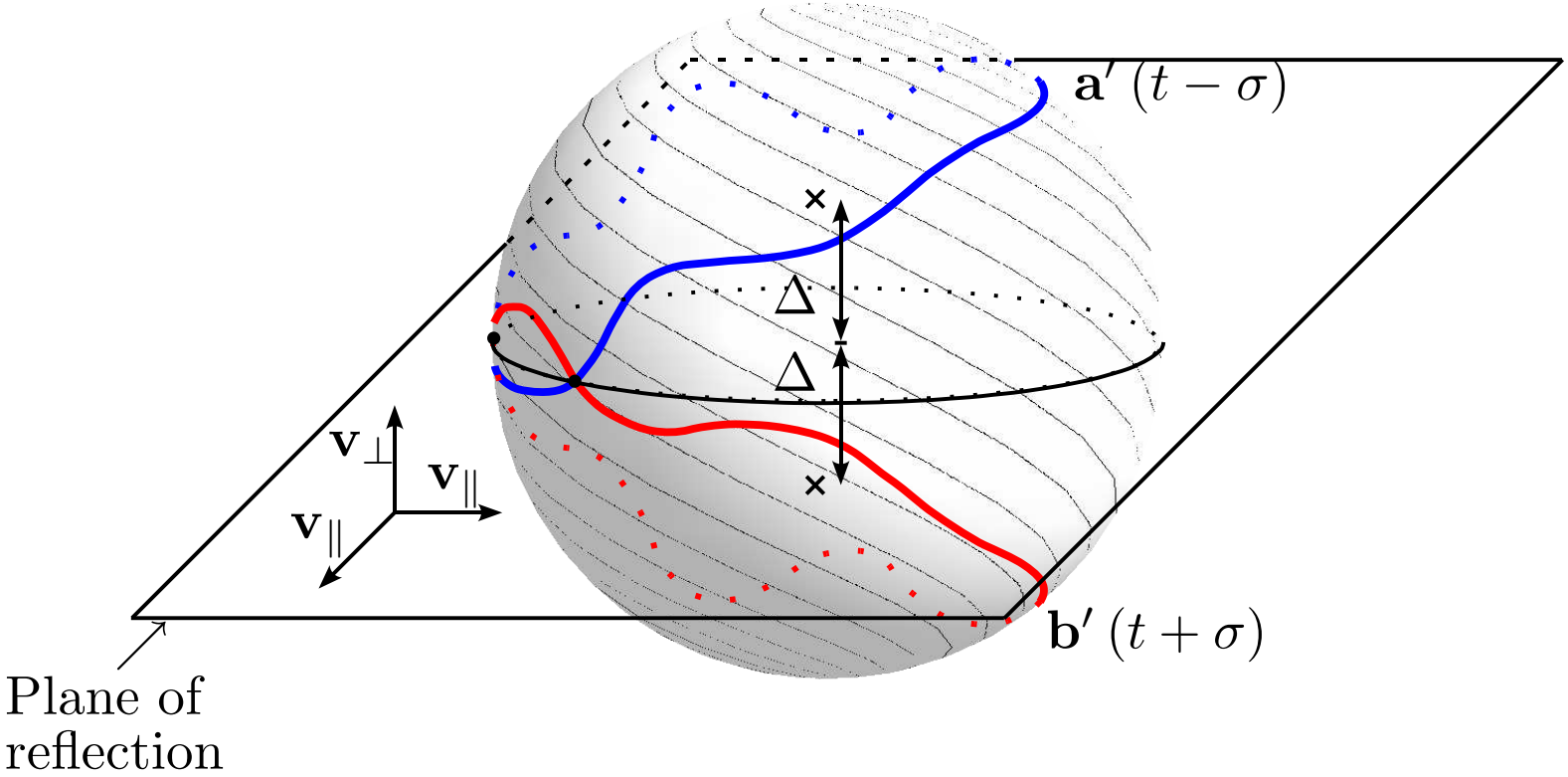

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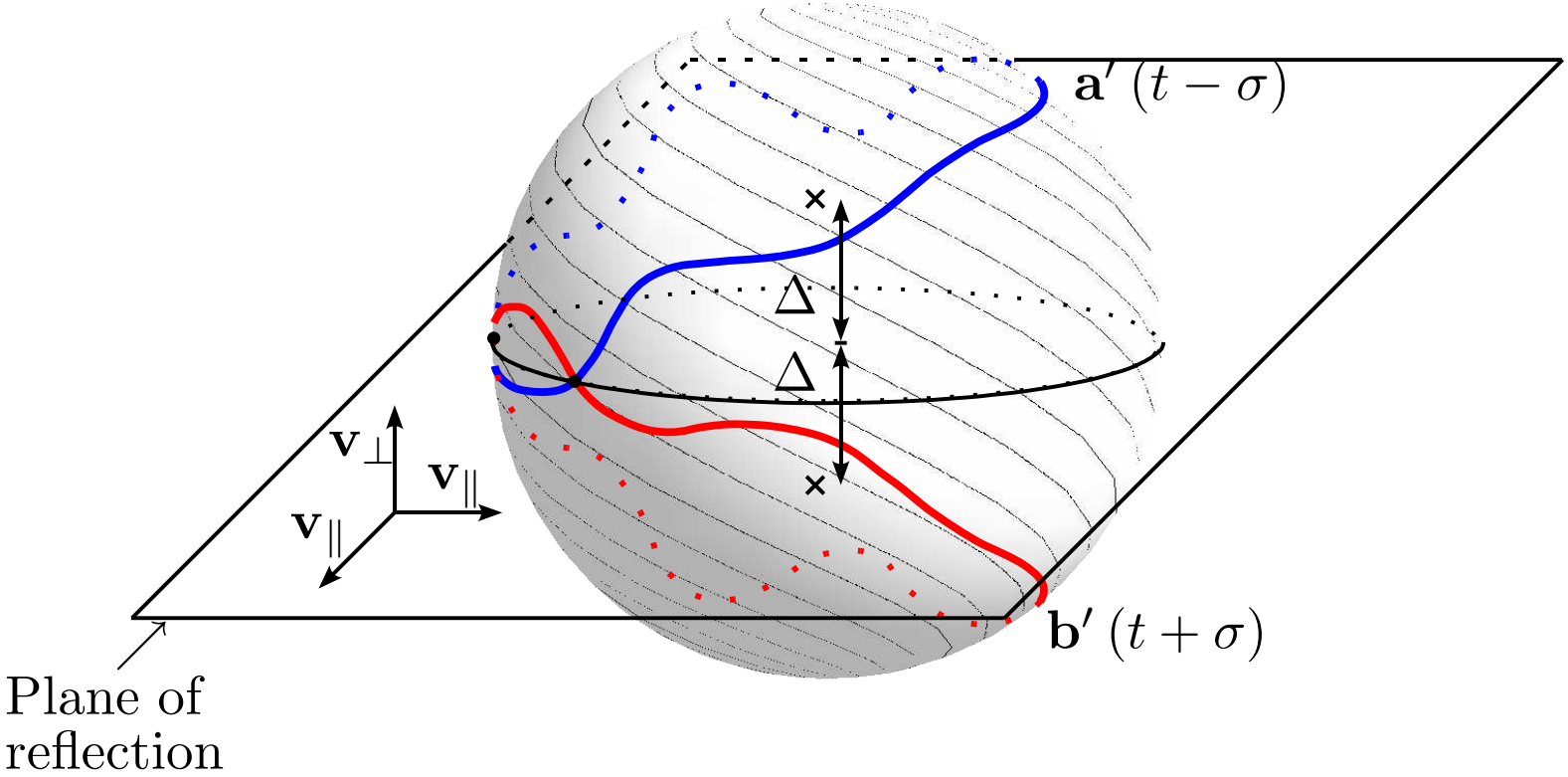

The closed curves will (typically) intersect the plane of reflection when Δ is small.

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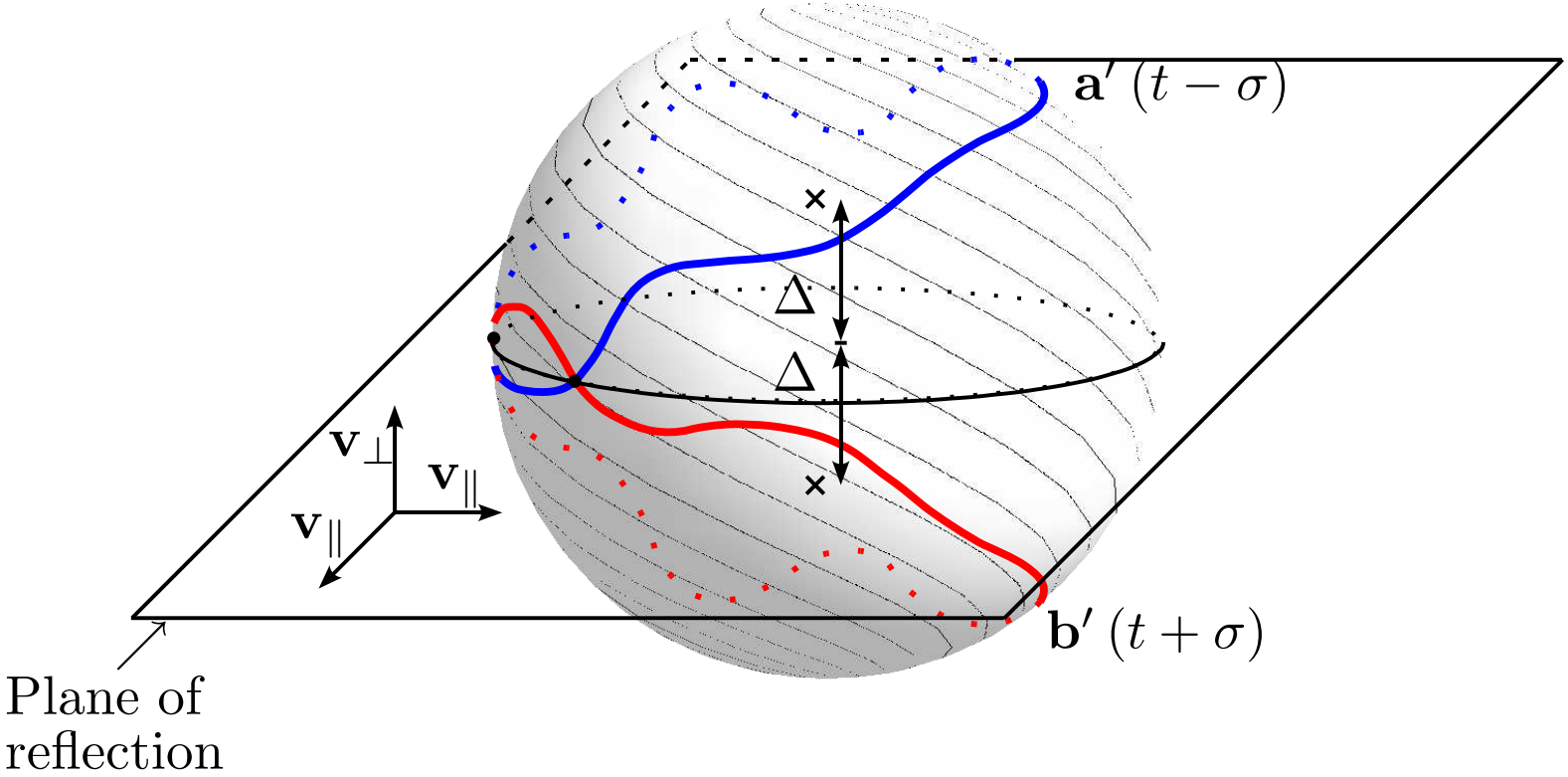

When the branes are close together, we generically get cusps

3 String Junctions

When standard Cosmic String collide, they intercommute.

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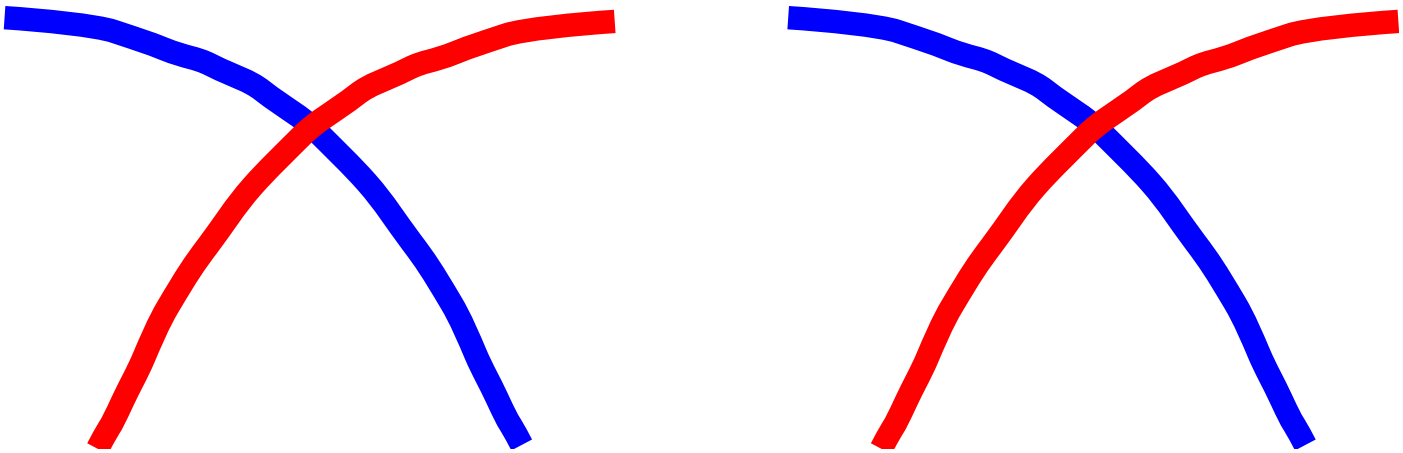

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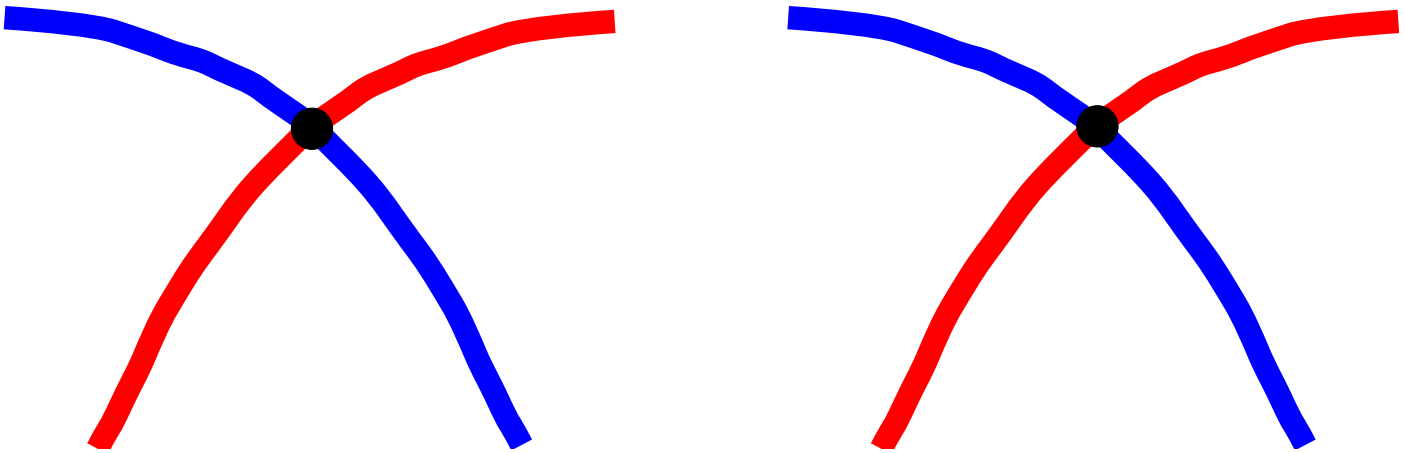

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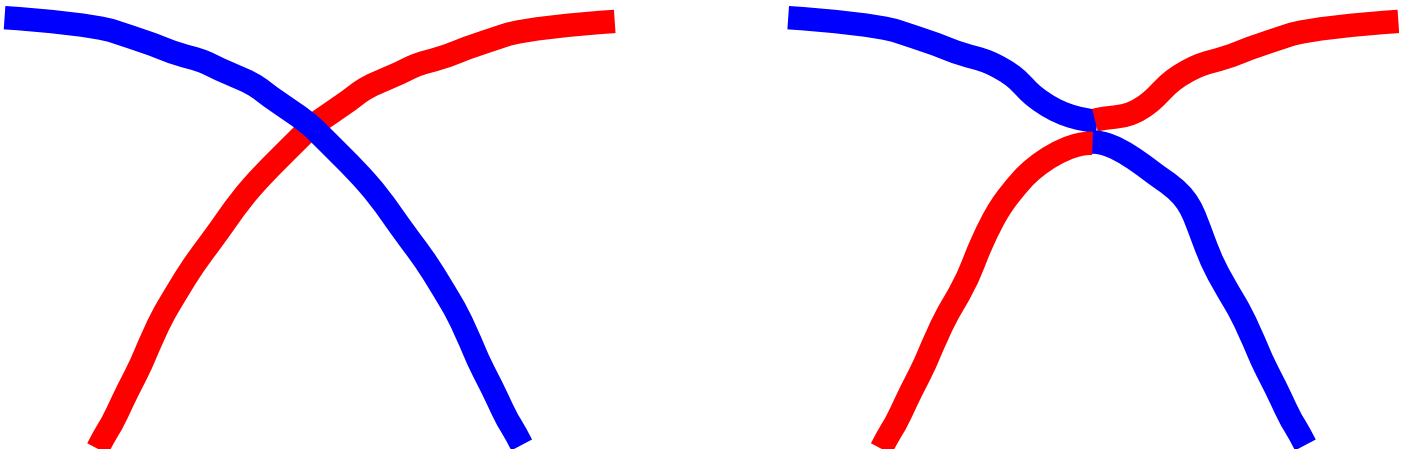

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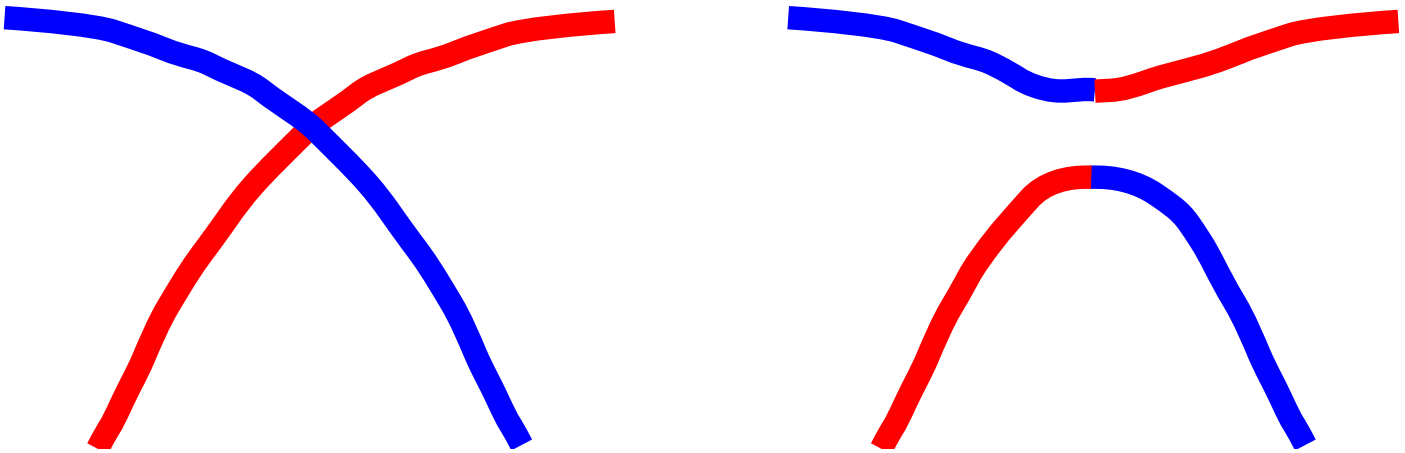

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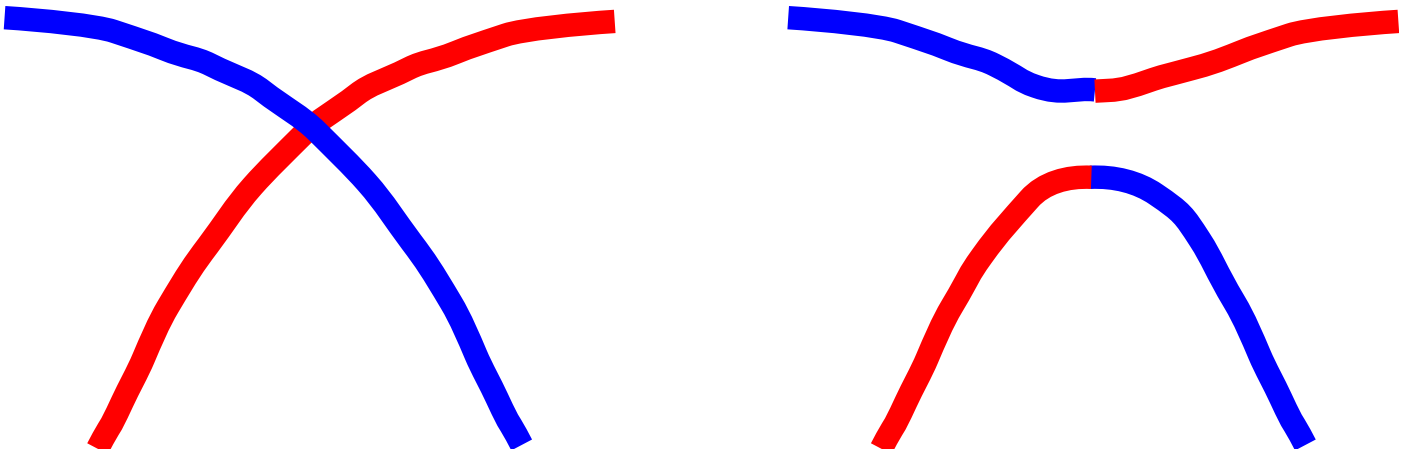

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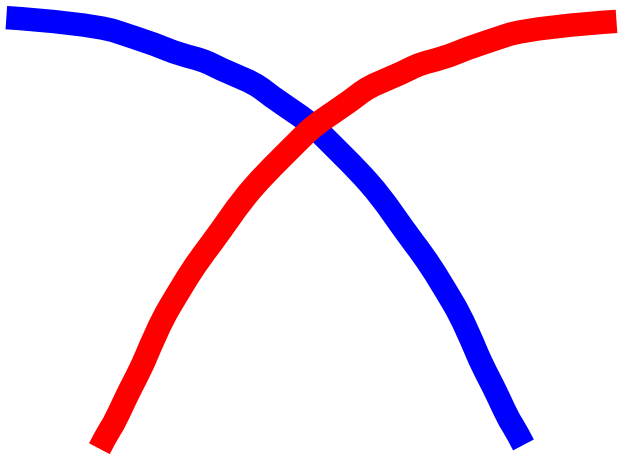

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The bound state can form because it is the BPS state for and F and D string.

M. Jackson, N. Jones, J. Polchinski (2005)

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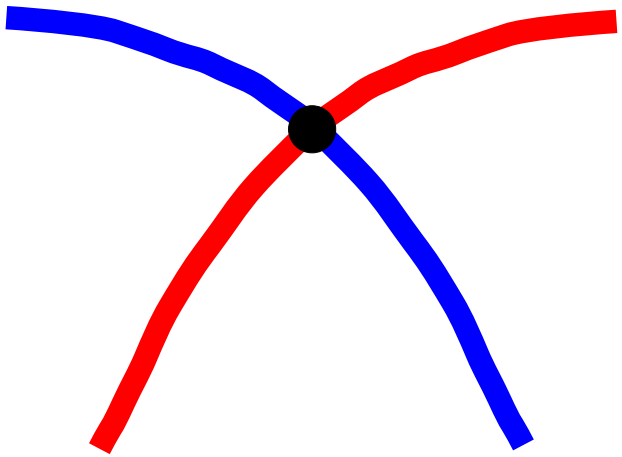

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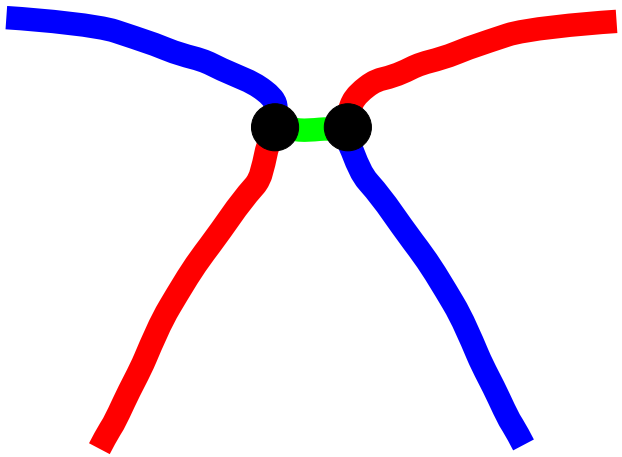

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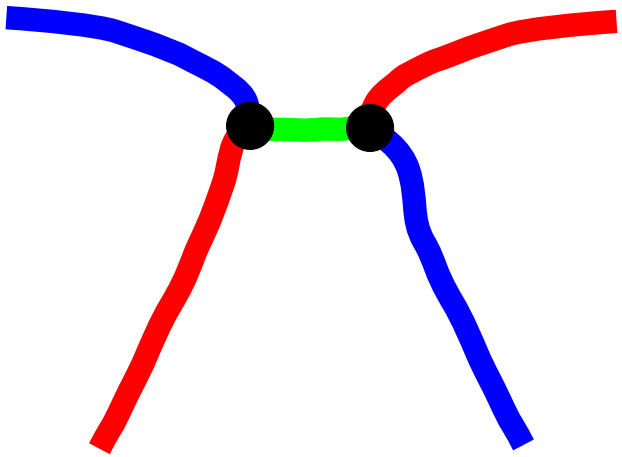

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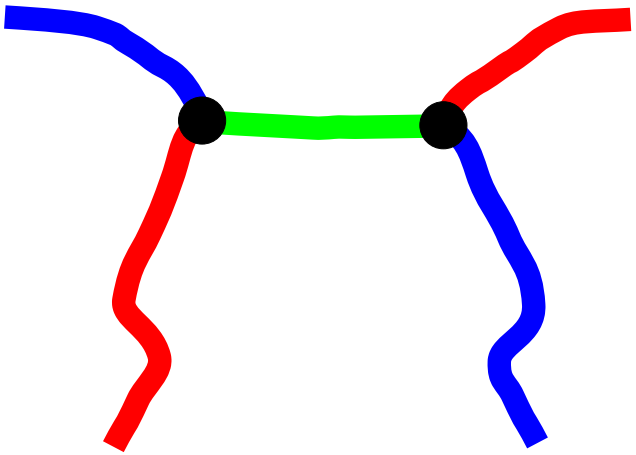

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3 String junction

Action: Copeland, Kibble, Steer (2006)

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\begin{aligned}
S &= -\sum_i \mu_i \int \mathrm{d}\tau \int_0^{L_i(\tau)} \mathrm{d}\sigma \sqrt{-x_i'^2 \dot{x}_i^2 - (F^i_{\tau\sigma})^2} \\
&\quad + \sum_i \int \mathrm{d}\tau \Big\{ \mathbf{f}_i(\tau) \cdot [x_i(\tau, L_i(\tau)) - \bar{x}(\tau)] \\
&\quad + g_i(\tau) \left[A^i_\tau(\tau, L_i(\tau)) + \dot{L}_i A^i_\sigma(\tau, L_i(\tau)) - \bar{A}(\tau) \right] \Big\} ,
\end{aligned}
$$

$\bar{x}(\tau)$ is the position of the junction,
$\bar{A}(\tau)$ is the electric field at the junction.

But note that this is still taking the infinitely thin limit i.e. it does not account for the microscopic structure of the Superstrings.

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For each string the equations of motion are the same,

$$\begin{aligned} \ddot{\mathbf{x}}_i - \mathbf{x}_i'' &= 0 \,, \\ \partial_\tau p_i = \partial_\sigma p_i &= 0, \end{aligned}$$

$$\text{where } p_i = \frac{\lambda^2 \mu_i F^i_{\tau\sigma}}{\sqrt{-x_i'^2 \dot{x}_i^2 - \lambda \left(F^i_{\tau\sigma}\right)^2}} \,.$$

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3 String Junction

We set up the strings so that the $\sigma_i = 0$ ends are attached to parallel Dn-branes, so the junction is at the $\sigma_i = L_i(\tau)$ end of each string.

The Lagrange multipliers give three equations:

$$[\bar{\mu}_1 + \bar{\mu}_2 + \bar{\mu}_3]\left[1 + \dot{L}_1\right]\mathbf{b}_1'(t + L_1(t)) =$$
$$[\bar{\mu}_1 - \bar{\mu}_2 - \bar{\mu}_3]\left[1 - \dot{L}_1\right]\mathbf{a}_1'(t - L_1(t))$$
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and cyclic permutations.
where the effective string tension is defined as

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Cosmic Superstring Junctions

This holds for any three DBI-strings.

For Cosmic Superstrings we have a relation between the tensions.

$$\bar{\mu}_1 = 1 \,, \quad \bar{\mu}_2 = \frac{1}{g_s} \,, \quad \bar{\mu}_3 = \sqrt{1 + \frac{1}{g_s^2}} = \frac{1}{g_s} + \frac{g_s}{2} + \mathcal{O}\left(g_s^3\right) \,,$$

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Using this expansion, the junction boundary condition become:

$$
\begin{aligned}
(S_{23} - 2S_{13} - 2S_{12})\,\mathbf{b}'_1 &= S_{23}\mathbf{a}'_1 - 2S_{13}\mathbf{a}'_2 - 2S_{12}\mathbf{a}'_3\ , \\
\mathbf{b}'_2 &= \mathbf{a}'_3\ , \\
\mathbf{b}'_3 &= \mathbf{a}'_2\ ,
\end{aligned}
$$

and the velocities become

$$
\dot{L}_1 = 1 - \frac{S_{23}}{S_{12} + S_{13}} \quad , \dot{L}_2 = \frac{S_{12} - S_{13}}{S_{12} + S_{13}}\ , \quad \dot{L}_3 = \frac{S_{13} - S_{12}}{S_{12} + S_{13}}\ ,
$$

where $S_{ij} = \frac{1}{2}(1 - \mathbf{a}'_i \cdot \mathbf{a}'_j)$.

Cosmic Superstring Junctions

D and FD strings (strings 2 and 3)

$$\begin{aligned} \mathbf{b}_2' &= \mathbf{a}_3' \,, \\ \mathbf{b}_3' &= \mathbf{a}_2' \,, \end{aligned} \qquad \dot{L}_2 = -\dot{L}_3 \,.$$

in-movers on string 2 become out-movers on string 3 and vice versa.

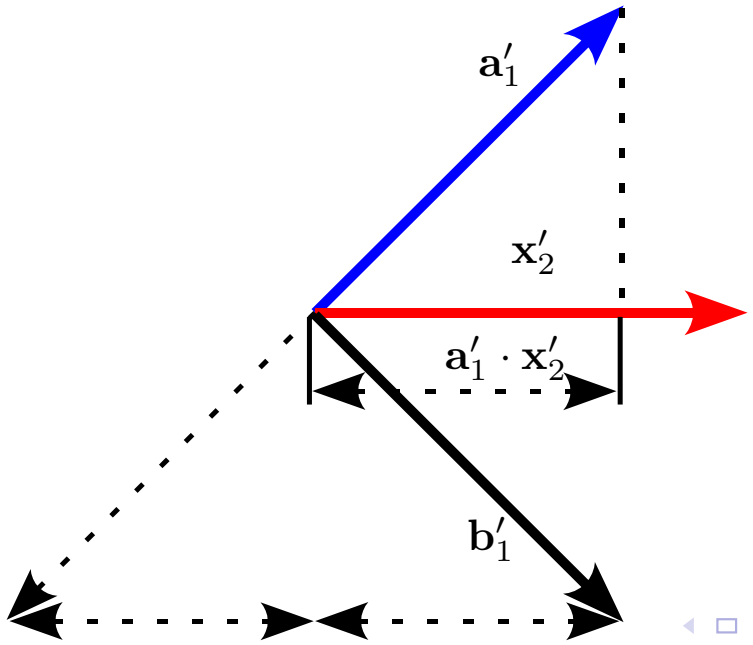
i.e. the (heavy) D-string and the (heavy) bound state string, behave as one long string that is unaffected by the presence of the (light) F-string.

Cosmic Superstring Junctions

Consider the limit where the (heavy) D/FD-string moves slowly, $|\dot{\mathbf{x}}_2| \ll 1$.

F-string (string 1)

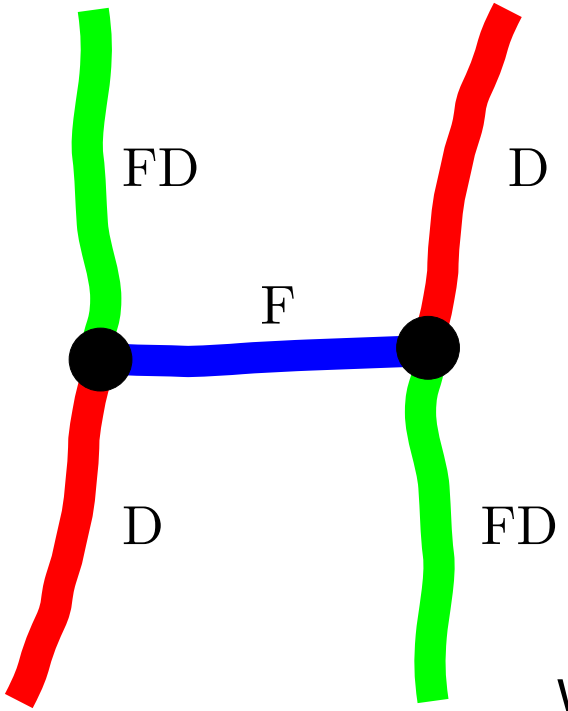
$$\mathbf{b}_1' \approx \frac{2\left(\mathbf{a}_1' \cdot \mathbf{x}_2'\right)\mathbf{x}_2' - |\mathbf{x}_2'|^2 \mathbf{a}_1'}{2 - |\mathbf{x}_2'|^2}, \quad \dot{L}_1 \approx 0$$



Cosmic Superstring Junctions

So the junction looks exactly like a standard D1-brane boundary condition for the (light) F-string.

We have considered the $\sigma_i = 0$ ends of the string being attached to D-branes, and now we see that these could just as easily be another junction.



will have cusps, on the F string.

Cosmic Superstring Junctions

When $|\dot{\mathbf{x}}_2|$ is not small, there is an additional rotation of the closed curve $\mathbf{b}_1'$.

Because the intersection of the closed curves $\mathbf{a}_1'$ and $\mathbf{b}_1'$ is a topological property, for small rotations we wil still get cusps.

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Conclusions

- Just as for Cosmic Strings, the solutions of DBI strings with junctions can be visualised by considering closed curves on a unit sphere.
- The boundary condition at three string junctions is an intermediate between Dirichlet and Neumann boundary conditions.
- Non-periodic boundary conditions can lead to cusps.
- In particular Cosmic Superstrings junctions provides such boundary conditions, opening up a new energy loss mechanism for the network, in addition to the energy loss from loops.
- To order g_s the dynamics of the 'heavy' strings in this network are unaffected by the presence of these junctions: Should still have scaling.

M. Hindmarsh, P. Saffi n (2006), A. Rajanite, M. Sakellariadou, H. Stoica (2007),

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Thanks for Listening

ANY QUESTIONS?