# Inhomogeneities in the Universe

# with exact solutions of General Relativity


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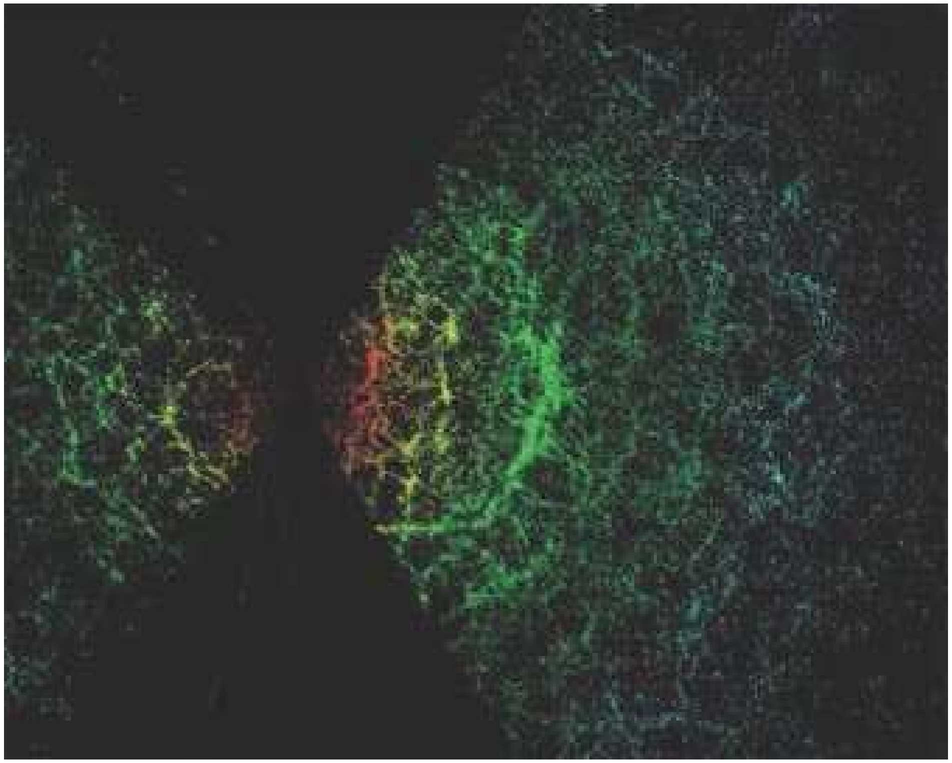
# The inhomogeneous Universe



Sloan Digital Sky Survey (false colors)

The Universe is not homogeneous. We see voids, groups of galaxies, clusters, superclusters, walls, filaments, etc. However, it is usually argued it should be nearly homogeneous at large scales (thus Friedmannian models); but how large are these scales and what does nearly imply?

# Methods to take inhomogeneity into account

- **Linear perturbation theory:** valid when **both** the curvature and density contrasts remain small. Not the case in the non-linear regime of structure formation and where the SNe Ia are observed.

- **Averaging methods "à la Buchert":** **promising** but needing to be improved.

- **Exact inhomogeneous solutions:** valid **at all scales.**, exact perturbations of the Friedmann background which they can reproduce as a limit **with any precision.**

# Exact inhomogeneous solutions used in astrophysics and cosmology

- **Lemaître – Tolman (L–T) models:** spherically symmetric dust solution of Einstein's equations. Determined by 1 coordinate choice + 2 free functions of the radial coordinate. FLRW is one subcase.

- **Lemaître models (usually known as Misner-Sharp)** – not an explicit solution but a metric determined by a set of 2 differential equations: spherically symmetric perfect fluid with pressure gradient.

- **Quasispherical Szekeres models:** dust solution of Einstein's equations with no symmetry. Defined by 1 coordinate choice + 5 free functions of the radial coordinate. L–T and FLRW are subcases.

- **Spherically symmetric Stephani models:** homogeneous-energy density, inhomogeneous-pressure solutions.

# Structure evolution with L–T models

- **Rich galaxy cluster:** A pure velocity perturbation can nearly produce a galaxy cluster. A mere density perturbation fails to do it. Velocity perturbations generate structures much more efficiently than density perturbations Krasiński and Hellaby 2004.

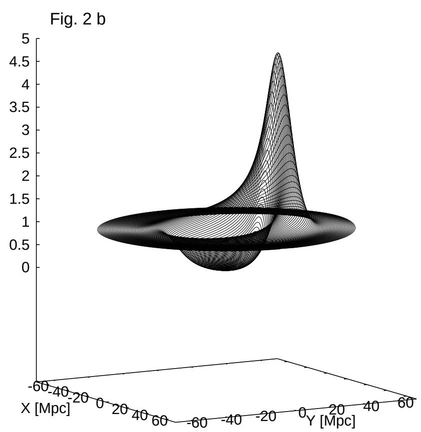
- **Void:** A void consistent with observational data (density contrast less than $\delta = -0.94$, smooth edges and high density in the surrounding regions) is very hard to obtain with L–T models without shell crossing Bolejko, Krasiński and Hellaby 2005.

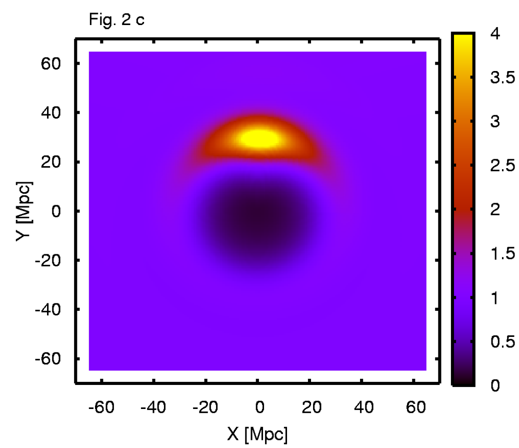
- Adding a realistic distribution of radiation (using Tolman models) helps forming such voids Bolejko 2006.

# Structure evolution with Szekeres models

## Double structure evolution

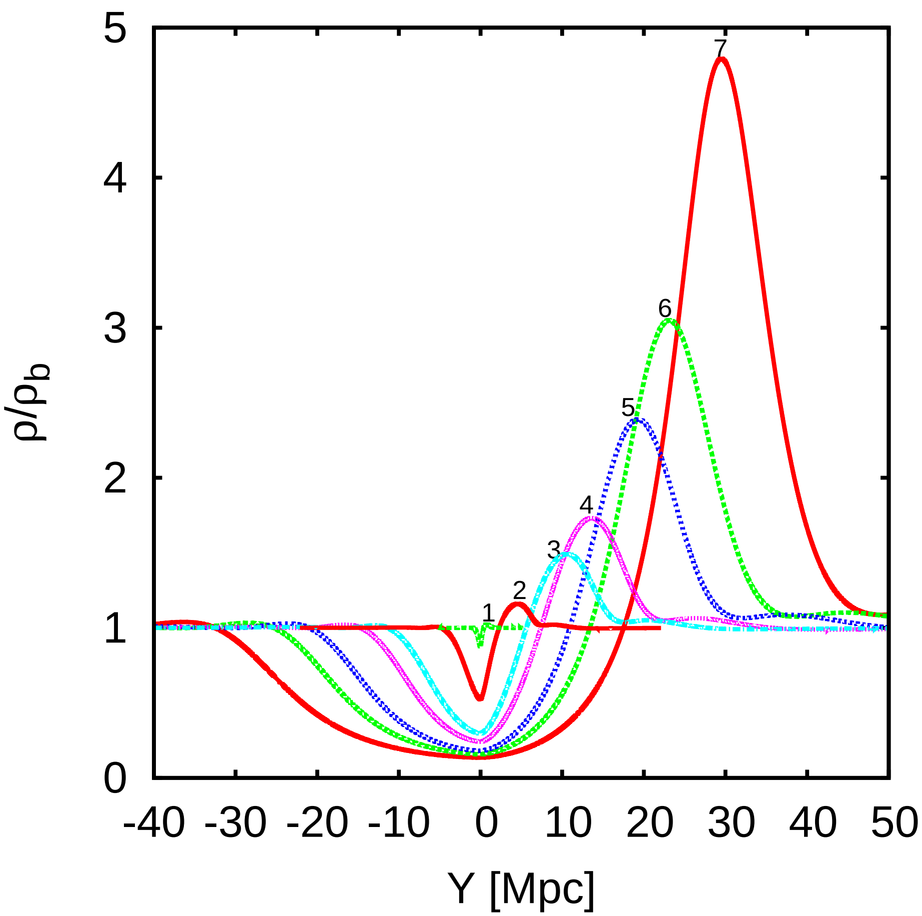
**Model of a void with an adjourning supercluster evolved inside an homogeneous background Bolejko 2006.**




**Current density distribution in background units.**

**To estimate how two neighbouring structures influence each other, the evolution of a double structure in QSS models has been compared with that of single structures in L–T models and linear perturbation theory.**

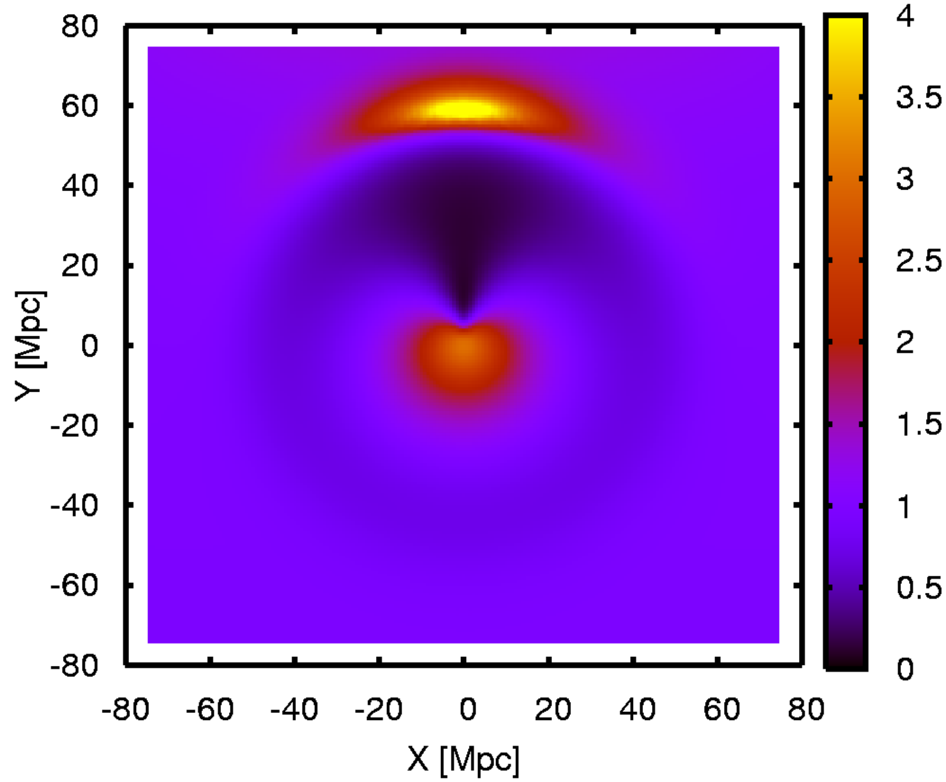



**Evolution of the density profile from 100 Myr after the Big Bang (1) up to the present time (7).**

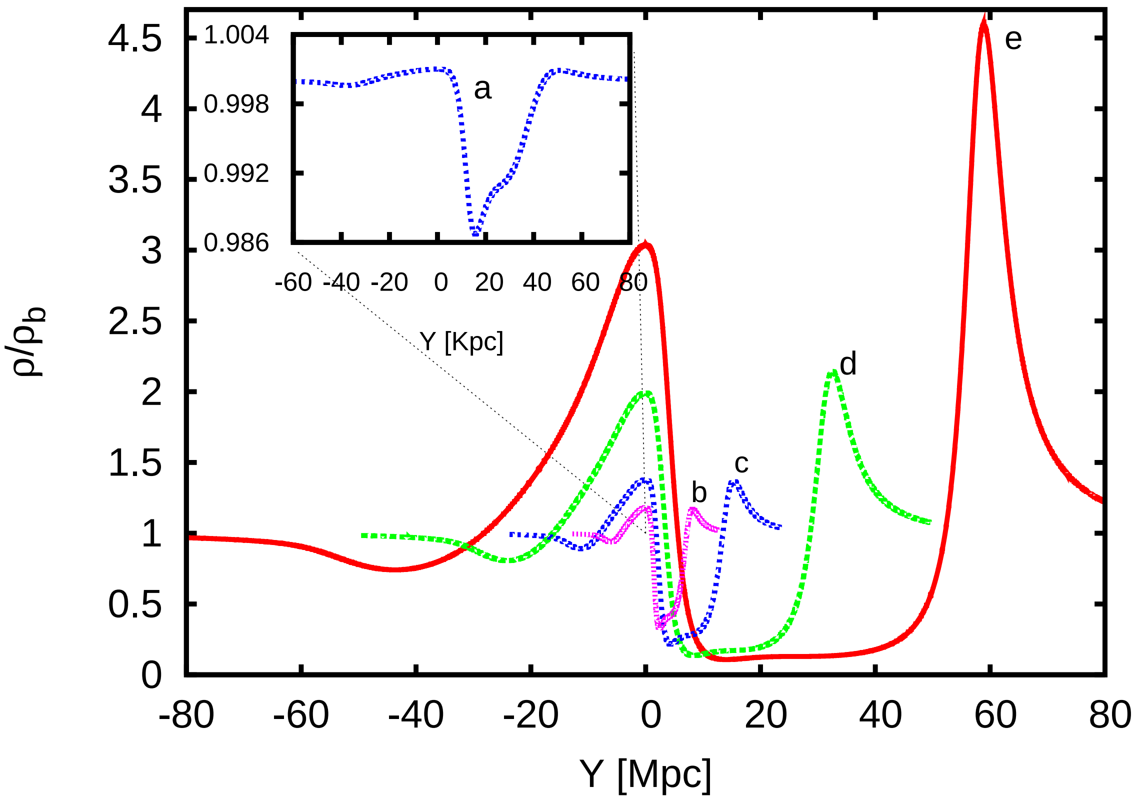
**In the QSS models studied, the growth of the density contrast is 5 times faster than in L–T models and 8 times faster than in the linear approach.**

# Triple structure evolution

The model is composed of an overdense region at the origin, followed by a small void which spreads to a given r coordinate. At a larger distance from the origin, the void is huge and its larger side is adjacent to an overdense region Bolejko 2007.




**Current density distribution.**




**Evolution of the density profile from 0.5 Myr after the Big Bang (a) up o the present time (e).**

**Where the void is large, it evolves much faster than the underdense region closer to the "centered" cluster. The exterior overdense region close to the void along a large area evolves much faster than the more compact supercluster at the centre. This suggests that, in the Universe, small voids surrounded by large high densities evolve much more slowly than large isolated voids.**

# Accelerated expansion: a mirage?

**What we see is not an accelerated expansion (this is only the outcome of the Friedmannian assumption) but the "dimming" of the supernovae, or more exactly their luminosity distance-redshift relation, itself infered from their light curve (flux measurements).**

**However, the acceleration interpretation was sufficiently misleading such as to induce some authors to try to derive or rule out no-go theorems, i.e., theorems stating that a locally defined expansion cannot be accelerating in models satisfying the strong energy condition. But this is not the point.**

**Other stressed, more accurately, that the definition of a deceleration parameter in an inhomogeneous model is tricky Hirata and Seljak 2005, Apostopoulos et al. 2006 and has nothing to do with reproducing the supernova data Krasiński, Hellaby, MNC, Bolejko 2009.**

# Use of exact inhomogeneous models in cosmology

L–T models have been most widely used as exact inhomogeneous models in cosmology since they are the most tractable among the few available (but QSS models are currently slightly coming into play).

But caution with L–T is required since:

- An origin, or centre of spherical symmetry, occurs at $r = r_c$ where $R(t, r_c) = 0$ for all $t$. The conditions for a regular centre were derived by Mustapha and Hellaby 2001.
- Shell crossings, where a constant $r$ shell collides with its neighbour, create undesirable singularities where the density diverges and changes sign. The conditions on the 3 arbitrary functions that ensure none be present anywhere in an L–T model were given by Hellaby and Lake 1985.
- The assumption of central observer, generally retained for simplicity, can be considered as grounded on the observed quasi-isotropy of the CMB temperature, and thus as a good working approximation at large scales. At smaller scales, it gives simplified models of the Universe averaged only over the angular coordinates around the observer, i. e., with the relax of only one degree of symmetry as regards the homogeneity assumption.

However, models assuming an off-centre observer and L–T Swiss-cheeses have also been studied to get rid of possible misleading features of spherical symmetry.

# Degeneracy of L–T models

It is well-known, from the work of Mustapha, Hellaby and Ellis 1997, that an infinite class of L–T models can fit a given set of observations isotropic around the observer. This has been confirmed by MNC 2000 while studying the fitting of L–T models with a central observer to the supernova data.

The problem of finding THE spherically symmetric model able to mimic dark energy is therefore completely degenerate. It is the reason why many different central observer L–T models have been proposed and shown to do the job rather well.

Thus, to constrain the model further on, it is mandatory to fit it to other cosmological data.

# Direct and inverse problem

**Two procedures for trying to explain away dark energy with (L–T) models:**

- **The direct way (smaller number of degrees of freedom than allowed): first guess the form of the parameter functions defining a class of models supposed to represent our Universe with no cosmological constant, and write the dependence of these functions in terms of a limited number of constant parameters; then fit these constant parameters to the observed SN Ia data or to the luminosity distance-redshift relation of the standard $\Lambda$CDM model.**

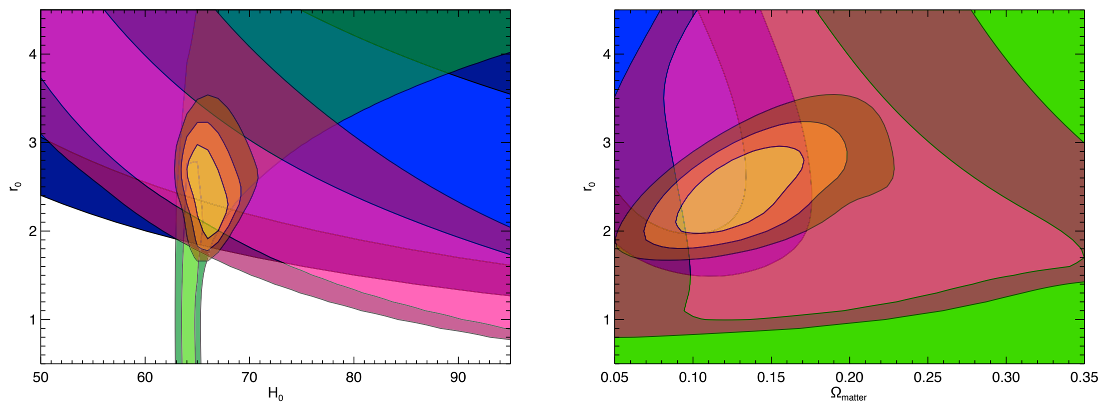
- **The inverse problem (more general): consider the luminosity distance $D_L(z)$ as given by observations or by the $\Lambda$CDM model as an input and try to select a specific L–T model with zero cosmological constant best fitting this relation.**

**Then, to avoid degeneracy, jump to a further step and try to reproduce more and possibly all the available observational data, Alnes et al. 2006, Bolejko 2008, Garcia-Bellido and Haugbolle 2008, Hellaby et al. 2007, 2008.**

# Example of central observer L–T: the GBH model

The GBH void class of L–T model (Garcia-Bellido and Haugbolle 2008a,b) is specified by its matter content $\Omega_M(r)$ and its expansion rate $H(r)$, governed by 5 free constant parameters and it matches to an E-deS universe at large scales.

These models are fitted to a series of observations (CMB, LSS, BAO, SN Ia, HST measure of $H_0$, age of the globular clusters, gas fraction in clusters, kinematic SZ effect for 9 distant galaxy clusters) to constrain their parameters.

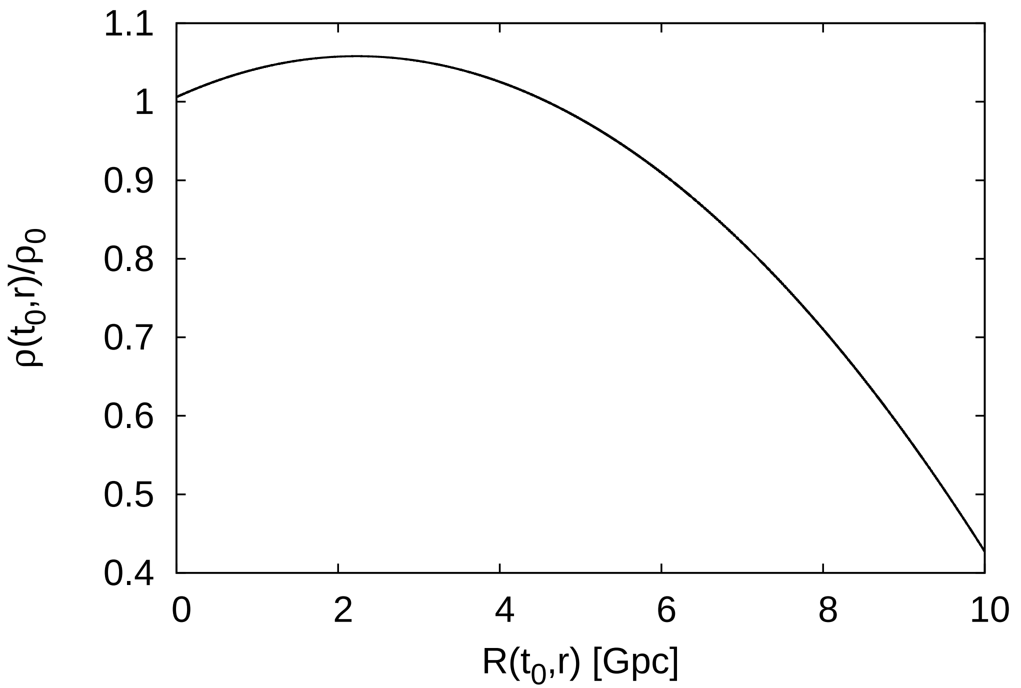



Two of the six likelihoods for the GBH constrained model: in yellow with 1-, 2-, 3-$\sigma$ contours = the likelihood for the combined data set; in blue, purple and green respectively with 1- and 2-$\sigma$ contours = the likelihood for the individual SN Ia, BAO and CMB data sets.

These models do not exclude the possibility that we leave close to the centre of a large (around 2.5 Gpc) void within an Einstein-de Sitter Universe, i. e., no dark energy.

## We can do without the giant void in a central observer L–T model

**Using a set of data – the angular diameter distance together with the galaxy number counts – both assumed to have the same form on our past light cone as in the $\Lambda$CDM model, the two left arbitrary L–T functions are determined and give the mass distribution in spacetime.**




**The current density profile does not exhibit a giant void but a giant hump. We are located in a shallow and wide funnel on top of this hump MNC, Bolejko, Krasínski, Hellaby 2009.**

**However, as the GBH giant void, this hump is not observable: it exists in the space t=now of events simultaneous with our current instant. Thus, even more, and more detailed observations will not be able to see them.**

**Why a hump and not a void? Here the functions determining the model are left arbitrary (no handpicked algebraic form containing a few constant parameters, no $t_B$ or E = const), hence no artificial constraints on the solution.**

## Example of L–T Swiss-cheese: Marra et al.'s model

**The model:** Lattice of L–T bubbles with radius $\geq$350 Mpc in E-de S background. Initially, the void at the center of each hole is dominated by negative curvature and a compensating overdensity matches smoothly the density and curvature EdS values at the border of the hole Marra et al. 2007.

Since the voids expand faster than the cheese, the overdense regions contract and become thin shells at the borders of the bubbles while underdense regions turn into emptier voids, eventually occupying most of the volume.

**Conclusion:** evolution of the voids, bends the photon paths and affects more photon physics than inhomogeneity geometry.

**Future prospects:** consider QSS models which exhibit enhanced structure evolution.

# Extracting the cosmic metric from observations

The inverse problem of deriving the arbitrary functions of a L–T model from observations is very much involved. This is the reason why most of the authors who have tried to deal with this issue have added some a priori constraints to the model Vanderveld et al. 2006, Chung and Romano 2006, Tanimoto and Nambu 2007.

LU and Hellaby 2007, McClure and Hellaby 2007 and Hellaby and Alfedeed 2009 initiated a program to extract the metric from observations. This is the full inverse problem and is not degenerate. To date it has assumed the metric has the L–T form, as a relatively simple case to start from, though the long term intention is to remove the assumption of spherical symmetry.

They developed and coded an algorithm that generates the L–T metric functions, given observational data on the redshifts, apparent luminosities or angular diameters, number counts of galaxies, estimates for the absolute luminosities or true diameters and source masses, as functions of $z$. This allows both of the physical functions of an L–T model to be determined without any a priori assumption on their form.

# Conclusions

- The increasing precision of observational data implies that FLRW models must now be considered just a zeroth order approximation, and linear perturbation theory a first order approximation whose domain of validity is an early, nearly homogeneous Universe.

- In the nonlinear regime, which was entered since structures formed, there is no escape from the use of exact methods (or of averaging schemes aiming at investigating this issue from the standpoint of backreaction).

- In the era of "precision cosmology", inhomogeneity effects on the determination of cosmological models cannot be ignored. Inhomogeneous models constitute an exact perturbation of the Friedmann background and can reproduce it as a limit with any precision.

- While using L–T models with a central observer, a giant void is not mandatory to explain away dark energy. A giant overdensity can also do the job. However neither the void nor the overdensity are observable. Don't try to see them on our past light cone.

- Exact inhomogeneous solutions can be employed not only for studying the geometry and dynamics of the Universe, but also to investigate the formation and evolution of structures.