An Empirical Gravity Law up to Galactic Scales

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Introduction

Looking for an empirically motivated modified gravity we start from the MOND proposal. Paying particular attention to the recent observational restrictions from local dSph galaxies.

MOND:

$$\mu(g/a_0)g = g_N$$

$$\mu(X) \rightarrow \begin{cases} 1 & X >> 1 \quad \therefore \quad g \rightarrow \frac{M}{R^2} \\ X & X << 1 \quad \therefore \quad g \rightarrow \frac{M^{1/2}}{R} \end{cases}$$

1) An explicit acceleration scale is introduced, $\boxed{a_0}$
2) A theory fully defined only at the limits.
$\mu(x)$ phases Newtonian term out as it phases Mondian term in.

Solar system dynamics favours an $\boxed{\text{abrupt transition function}}$, to get rid of the M term at solar system accelerations (e.g. Bekenstein 2004, PRD 70, 083509), which leads to a pure M term at galactic scales.

This in turn means you still need, $\boxed{\text{Dark Matter}}$ at cluster and dSph scales, e.g. Sanchez-Salcedo & Hernandez 2007, ApJ 667, 878

Need for DM in dSph's under MOND, with a sharp $\mu(X)$, as required by Solar System dynamics:

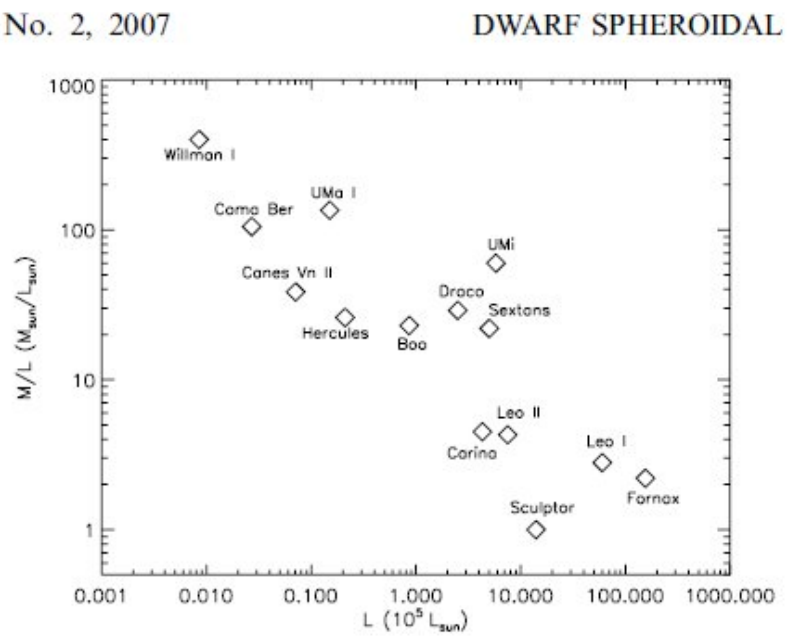

FIG. 2.—Inferred MOND mass-to-light ratios vs. luminosity for the known Galactic dSph galaxies at $40 < D < 260$ kpc and with velocity dispersion estimates (see text).

Normal stellar populations can reach M/L values of up to 10, but not beyond

Tidal heating has been suggested (e.g. Angus 2008, MNRAS 387, 1481) as an explanation, but troublesome as often no tidal extensions are evident, and tides tend to result in coherent velocity fields, not dynamical heating.

Ignoring for a minute Solar System restrictions, going back to original smooth transition $\mu(X)$, in particular:

$$\mu(X) = \frac{(1+4X)^{1/2} - 1}{(1+4X)^{1/2} + 1}$$

with which, the MOND acceleration can be written as:

$$g = g_N + g_M,$$

for a test particle at a distance R from a mass M yields,

$$g = -\frac{G_0 M}{R^2} - \frac{G_1 M^{1/2}}{R} = g_N(1+X)$$

$X = (G_1/G_0)(R/M^{1/2})$. For the standard value of $a_0 = 1 \times 10^{-8} cms^{-2}$ we get $G_1 = 1.2 \times 10^{-34} M_{\odot}^{-1/2} s^{-2} kpc^2$.

Written as a series, M term appears as a first correction, the following one would be a constant (dark energy?). Also, at dSph scales, N terms still plays a part, helpful in eliminating the need for DM.

Isothermal configurations

Taking advantage of the correspondence between hydrostatic equilibrium polytropic configurations and self gravitating stellar systems, we write the equation of hydrostatic equilibrium for a polytropic equation of state $p = K\rho^\gamma$,

$$K\gamma\rho^{\gamma-1}\frac{d\rho}{dr} = -\rho\nabla\phi \tag{1}$$

giving:

$$K\gamma\rho^{\gamma-2}\frac{d\rho}{dr} = -\frac{G_0 M(r)}{r^2} - \frac{G_1 M(r)^{1/2}}{r} \tag{2}$$

Changing ρ for $(4\pi r^2)^{-1} dM/dr$, and going to isothermal conditions, $K = \sigma^2$, $\gamma = 1$ yields:

$$\sigma^2\left[\left(\frac{dM}{dr}\right)^{-1}\frac{d^2M}{dr^2} - \frac{2}{r}\right] = -\frac{G_0 M(r)}{r^2} - \frac{G_1 M(r)^{1/2}}{r} \tag{3}$$

With σ the isotropic velocity dispersion for stars.

Taking initial conditions $\boxed{M(r) \to 0}$ and $\boxed{dM/dr = 4\pi r^2 \rho_0}$ for $r \to 0$, a constant central density ρ_0, we solve for $M(r)$ through a numerical scheme, with two input conditions, σ and ρ_0.

Isothermal configurations

Given the extra gravity, isothermal configurations have a finite total mass, and a well defined finite half mass radius.

σ and ρ_0 are fully determined by observational determinations.

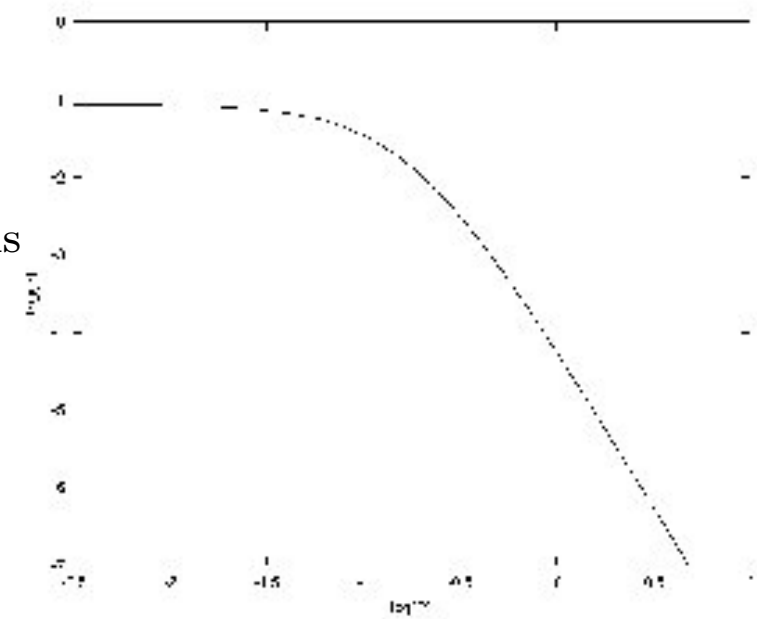

Figure 2. A sample isothermal equilibrium density profile, for $\rho_0 = 0.1 M_\odot pc^{-3}$ and $\sigma = 7 km/s$, resulting in a total mass of $3 \times 10^8 M_\odot$ and a volume half-mass radius of $0.39 kpc$. The density is measured in units of $M_\odot pc^{-3}$ and the distance in units of kpc.

Taking observed values of stellar σ for local dSph galaxies, isothermal configurations are constructed, and projected to obtain 2D half mass radii. Values of ρ_0 are varied to yield 2D half mass radii as given by observations, for each galaxy.

As models yield total mass, the observed luminosity then gives M/L values for the observed galaxies, under the gravity law being tested.

dSph M/L ratios obtained

Table 1. Basic properties and resulting M/L ratios for the sample of dSph galaxies.

Galaxy	$\sigma\,(km/s)$	$R_{hl}\,(kpc)$	$L_{tot} \times 10^5 L_\odot$	$(M/L)_A$	(M/L)	$< X >$	Age of Youngest Component (Gyr)
Carina	7 ± 1.8	0.290	4.3	$5.6^{+5.2}_{-2.9}$	$6.8^{+8.3}_{-4.6}$	7	3
Draco	8 ± 1.5	0.230	2.6	$43.9^{+29}_{19.3}$	$17.0^{+13.9}_{-8.9}$	4.1	10
LeoI	8 ± 1.2	0.330	48.0	$0.7^{0.65}_{0.3}$	$1.0^{+0.6}_{-0.44}$	5.7	2
Sextans	7 ± 1.0	0.630	5.0	$9.2^{+5.3}_{3.0}$	$6.3^{+4.2}_{-2.8}$	13.4	(2-6)
Fornax	12 ± 1.3	0.400	150.0	$1.4^{+0.45}_{-0.35}$	$1.4^{+0.6}_{-0.4}$	> 3.4	(2-3)
Sculptor	9.5 ± 1.7	0.160	22.0	$3.7^{+2.2}_{-1.4}$	$3.4^{+2.1}_{-0.7}$	> 2.2	> 5
LeoII	6 ± 1.4	0.185	7.0	$1.85^{+2}_{-1.1}$	$2.3^{+2.4}_{-04}$	5.5	6.5
Ursa Minor	8 ± 2	0.300	5.8	$5.8^{+6.5}_{3.6}$	$8.0^{+9.8}_{-5.1}$	> 5.3	12

$(M/L)_A$ gives the values for the mass to light ratios calculated by Angus (2008) under standard MOND, and (M/L) those in this study under the proposed gravity law. Total luminosities (in the V band) and half light radii are from Wilkinson et al. (2006), Wilkinson et al. (2004), Koch et al. (2007), Klenya et al. (2004), Walker et al. (2006), Mateo (1998), Coleman et al. (2007) and Irwin & Hatzidimitriou (1995), for the galaxies in the Table, in the order given, as summarised in Gilmore et al. (2007). Velocity dispersions are from adjusting a constant value to the data of Angus (2008). $< X >$ gives the average value of the parameter X defined in section (2), which measures the relative relevance of the Newtonian and MONDian terms in equation (6). The final column gives an estimate of the age of the youngest stellar population present in each of the systems, allowing an interpretation of the inferred (M/L) values in terms of stellar evolution, from Hernandez et al. (2000) for Carina, Ursa Minor, LeoI and LeoII, from Coleman et al. (2008) for Fornax, from Aparicio et al. (2001) for Draco, from Lee et al. (2003) for Sextans, and from Babusiaux et al. (2005) for Sculptor.

- Resulting M/L values are now fully consistent with old stellar populations, no need to add dark matter, or invoke extra dynamical effects

- A very interesting correspondence is apparent between the inferred M/L values and the age of the youngest component for each galaxy, natural in MOND, a peculiar coincidence in DM.

Ages of dSph stars

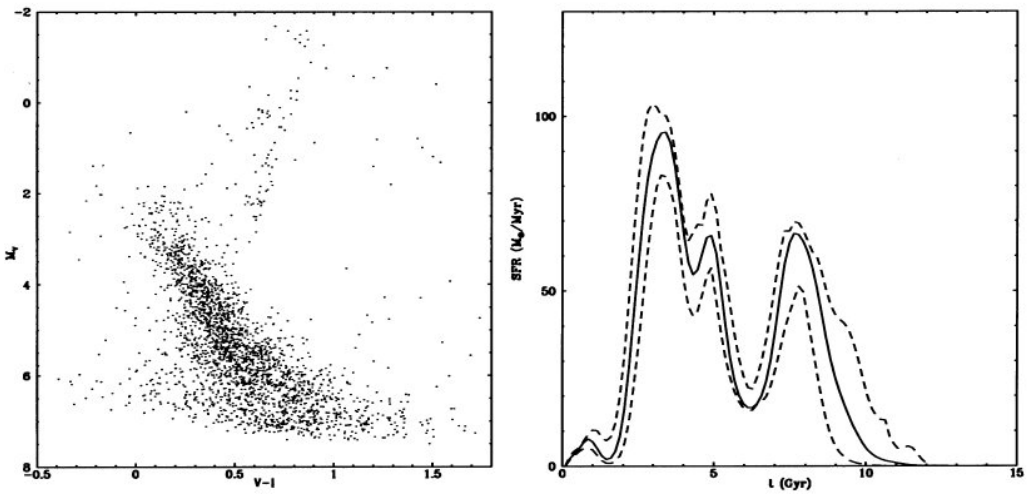

Figure 1. Left: observational HR diagram for Carina. Right: inferred *SFR*(*t*) for the central values of the observational parameters, solid line. The dashed curves represent the error envelope as defined by the quoted uncertainties in the foreground extinction and distance modulus.

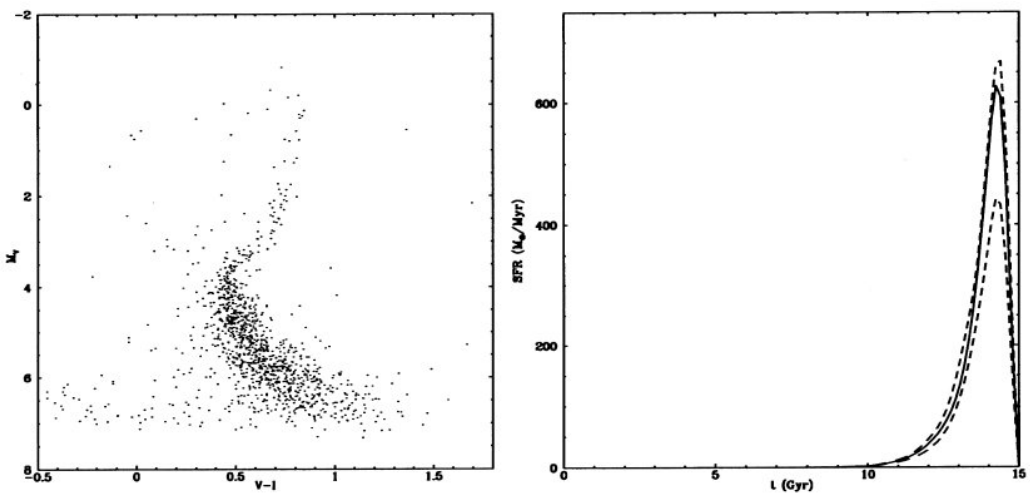

Figure 3. Left: observational HR diagram for Ursa Minor. Right: inferred *SFR*(*t*) for the central values of the observational parameters, solid line. The dashed curves represent the error envelope as defined by the quoted uncertainties in the foreground extinction and distance modulus.

Star formation histories now available through direct modelling of the observed resolved stellar populations.

e.g. Hernandez, Gilmore & Valls-Gabaud 2000, MNRAS 317, 831.

Internal structure of local dSph's

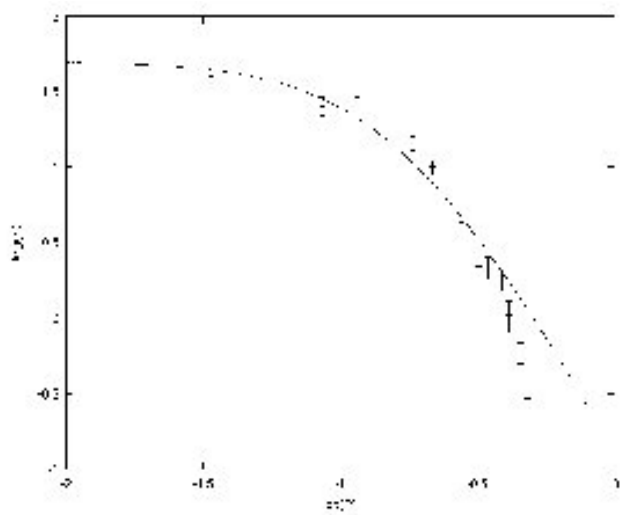

Figure 3. Comparison of our projected surface density profile for an equilibrium isothermal solution to equation (8), having input σ and projected R_{hm} as the observationally determined values for LeoII, dashed curve. For comparison, we give also the star counts surface density profile for Leo II of Coleman et al. (2007). Both profiles have been normalised to the same total luminosity.

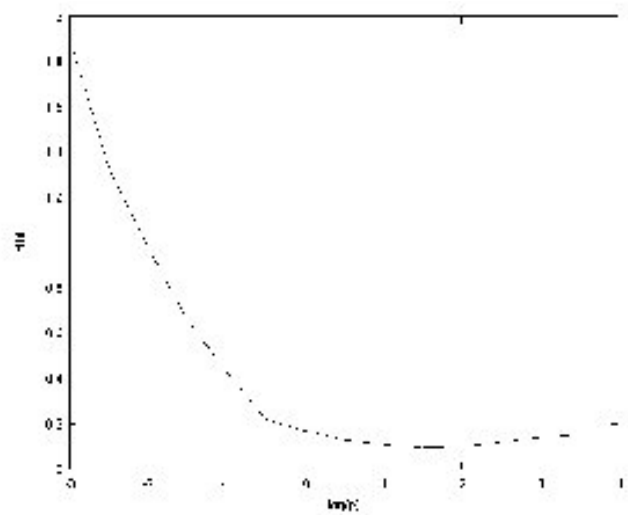

Figure 4. Dependence of the resulting projected half mass radius in kpc, against the assumed central density in $M_{\odot}pc^{-3}$, for isothermal equilibrium configurations under the proposed gravity law, at fixed $\sigma = 10km/s$. A very broad region where R_{hm} remains almost constant is evident, at precisely the level found to define the minimum R_{hl} values for observed dSph galaxies, systems having typical values of $\sigma \simeq 10km/s$

- 2D projected surface mass density profiles agree with observed surface brightness profiles.

- For a large range of ρ_0 and σ values, the resulting projected half mass radii lie within a narrow range of values, as do observed dSph galaxies.

-As expected from MOND, we obtain $M_{tot} = \left(\frac{5.5\sigma^2}{G_1}\right)^2$.

Local dSph Scalings

Assigned Newtonian M/L values show interesting scalings with observed luminosities, for local dSph galaxies.

Assigned M/L_N values will be given by:
$(M/L)_N = \left(\frac{10\sigma^2 R_{hl}}{G_0}\right)\left(\frac{1}{L_{tot}}\right)$

for R_{hl}=0.3 and replacing σ through
$M_{tot} = \left(\frac{5.5\sigma^2}{G_1}\right)^2$, taking an intrinsic stellar M/L=5 yields:

$$\boxed{(M/L)_N = \left(\frac{3\sqrt{5}}{5.5}\right)\left(\frac{G_1}{G_0}\right)\left(\frac{1}{L_{tot}}\right)^{1/2}}$$

Introducing $M_V = -2.5log(L_{tot}) + 4.83$ gives:

$$\boxed{log(M/L)_N = 3.77 + 0.2M_V.}$$

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$$log(M/L)_N = 3.77 + 0.2M_V.$$

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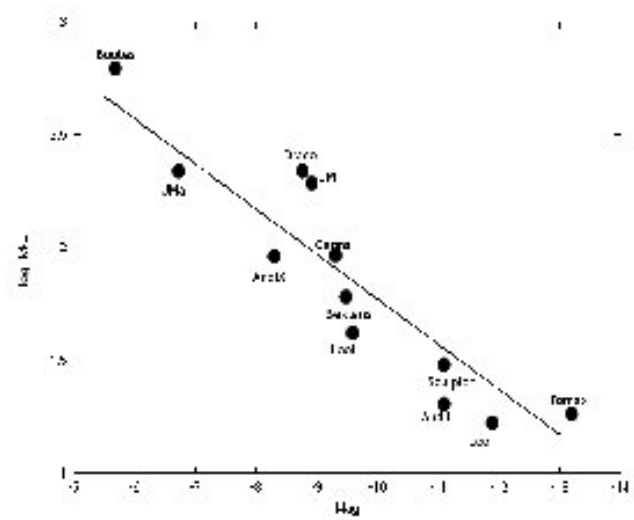

Figure 5. Logarithms of the Newtonian M/L values of local dSph galaxies, from Gilmore et al. (2007), against total observed V band magnitudes. The straight line gives the Newtonian M/L values which would be assigned to isothermal equilibrium populations of stars having a constant intrinsic stellar M/L ratio of 5, and a constant half mass radius of $300kpc$, consistent with what is observed, and to what the proposed model yields for equilibrium configurations having velocity dispersions of $10km/s$, as the observed dSph show, according to the gravity law presented here.

Conclusions:

One of the original smooth transition $\mu(X)$ functions of MOND can be written as a series expansion, in terms of an acceleration parameter.

• For isothermal equilibrium configurations for dSph galaxies, where MOND has been most controversial, we show agreement is improved, and no dark matter is needed. Surface density mass profiles are also in accordance with observed surface luminosity profiles

• The resulting M/L ratios are in qualitative accordance with the relative ages of the stellar populations of the individual dSph galaxies, for the sample studied. The observed scalings in R_{hl} and assigned $(M/L)_N$ values as a function of total magnitudes are explained naturally by the proposed gravity law.

• Such smooth transition $\mu(X)$ functions are inconsistent with Solar System dynamics

• An empirical gravity law up to galactic scales probably requires more than just MOND, e.g. the inclusion of an explicit distance scale e.g.:

$$g = g_N + g_N^m$$

where $m = 1 - \left(\frac{r}{2r+R_s}\right)$