Spherical cold collapse revisited

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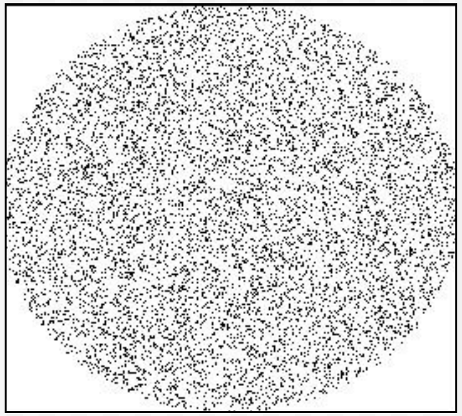
The problem: Cold Spherical Collapse & Virialization

N <u>point</u> particles randomly distributed in a sphere

Cold i.e. velocities=0

“Turn on” Newtonian gravity…



Motivations..

•“Simple” reference (one parameter) family of initial conditions for study of violent relaxation and virialisation

•Poses issues about discreteness (i.e. N-dependent) effects in an interesting way

•Study uncovered interesting and unexpected results….

Part I: finite systems

Cold collapse: theory

$N \to \infty$ limit: uniform density "spherical collapse model" (SCM)

Evolution is homologous rescaling $r(t) = R(t) r(0)$

R(t): Friedman equation!

Singularity ($R \to 0$) at finite time

$$\tau_{scm} \equiv \sqrt{\frac{3\pi}{32 G \rho_0}}$$

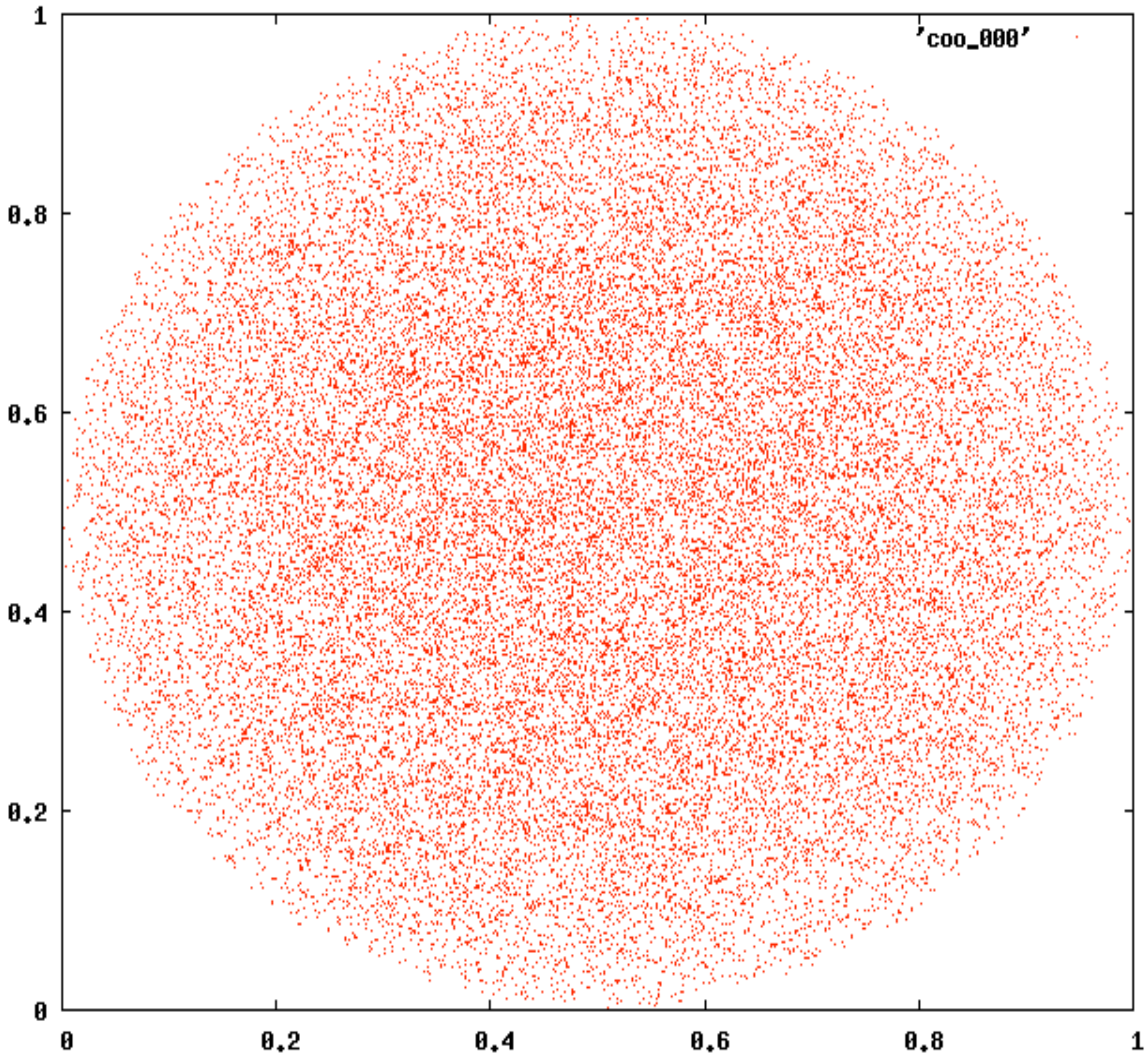
where ρ_0 is the mean mass density ($\to$ *independent of initial size*)

Cold collapse

Phenomenology at finite N …

Same phenomenology as for any such IC:

Collapse + violent relaxation → **virialized "equilibrium" state**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1

Questions…

•How is the singularity in collapse regulated at finite N?
cf. Aarseth et al. 1988, Boily et al 2002

Here focus on:

•How do properties of virialized state depend on N?

•Is the dynamics collisionless ($\leftrightarrow$ independent of N) ?

Numerical simulations

Range of N explored: few x 10^2 → few x 10^5

Simulate with GADGET
<u>Force is softened at a scale ε</u>

<u>Checks:</u>

- Energy conservation
- Convergence tests for all of numerical parameters, and **ε**
- Cross-check smaller N results with an N^2 code without smoothing

Note: we interpret our results as **representative of the limit ε→0**

Minimal size: theory

At any finite N, the singularity is regulated by the perturbations.

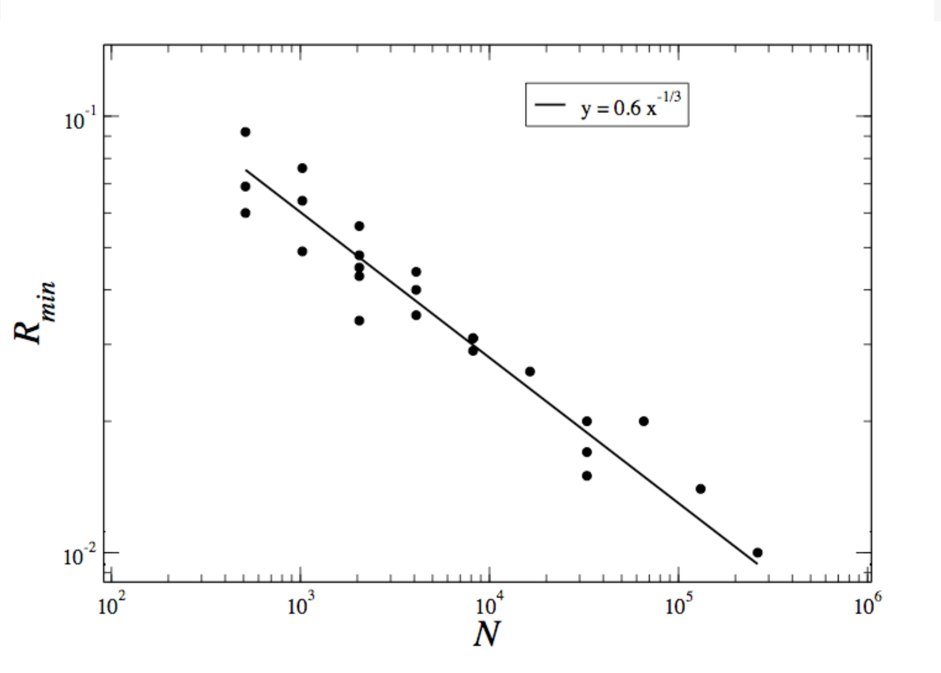
A **minimal collapse radius R^{min}** can be estimated analytically, assuming

- Perturbations are small and evolve as in fluid limit
- Collapse stops when perturbations reach some given amplitude

This gives

$$R_{min} \propto N^{-1/3}$$

Minimal size: phenomenology



In agreement with previous studies (Aarseth et al. 1988, Boily et al 2002) we find

$$R_{min} \propto N^{-1/3}$$

Cold collapse

The energy budget

Total initial energy E_0 is asymptotically sum of **three** non-zero energies:

$$E_0 = W^n + K^n + K^p$$

W^n: potential energy of **bound (negative energy)** particles
K^n: kinetic energy of **bound (negative energy)** particles
K^p: kinetic energy of **ejected (positive energy)** particles

Further virialization implies $$2K^n + W^n = 0$$

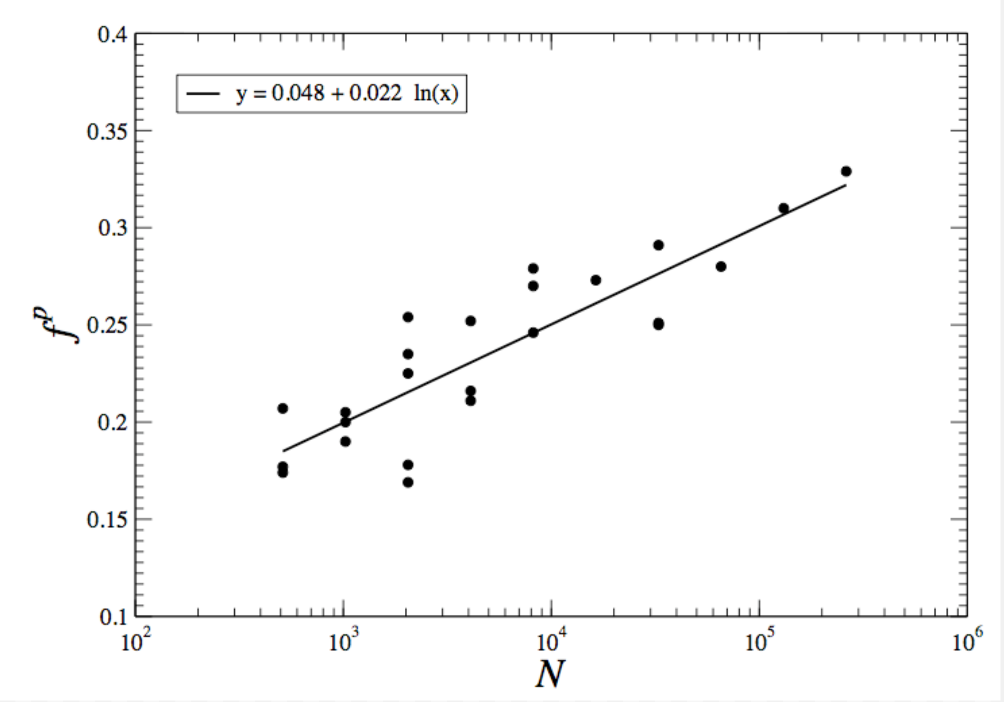
Thus **one unknown** - choose K^p.

For mass likewise, we define f^p= **fraction of mass with positive energy**

Question: Do f^p and K^p depend on N?

Cold collapse

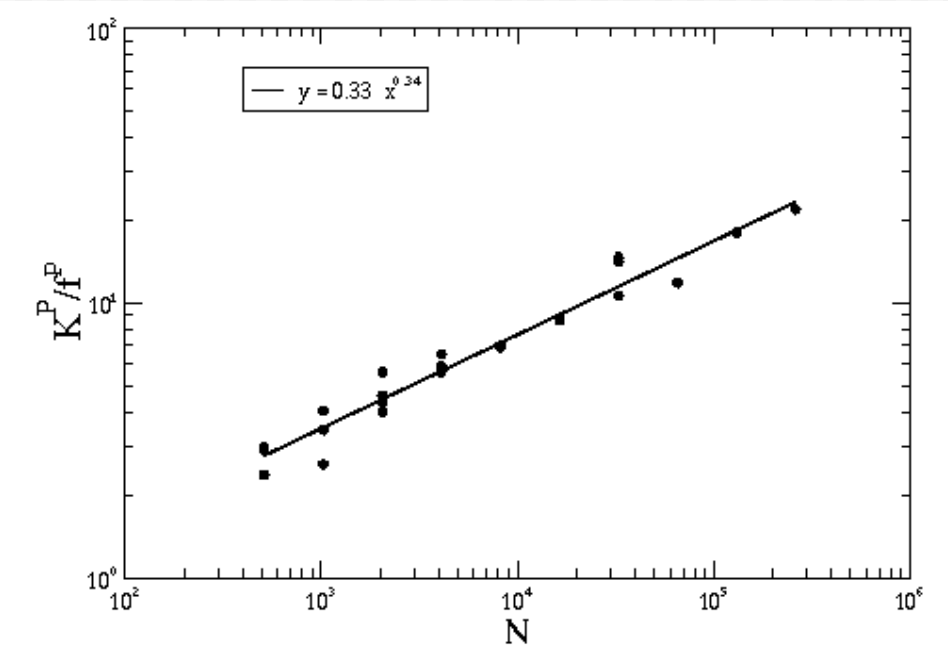
Mass ejection



Slow (approx logarithmic) increase in ejected mass as a function of N

Cold collapse

Energy ejection

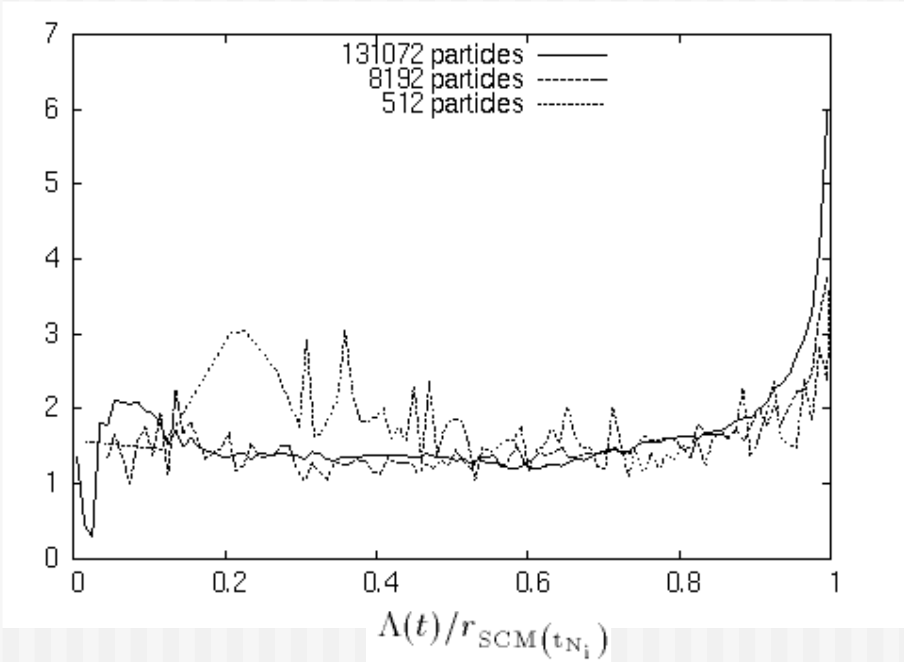


$$K^p / f^p \propto N^{1/3}$$

Cold collapse

Mechanism of energy ejection

Detailed study of collapse phase reveals mechanism of ejection:
Outer particles lag (compared to SCM) on average, “arrive” late

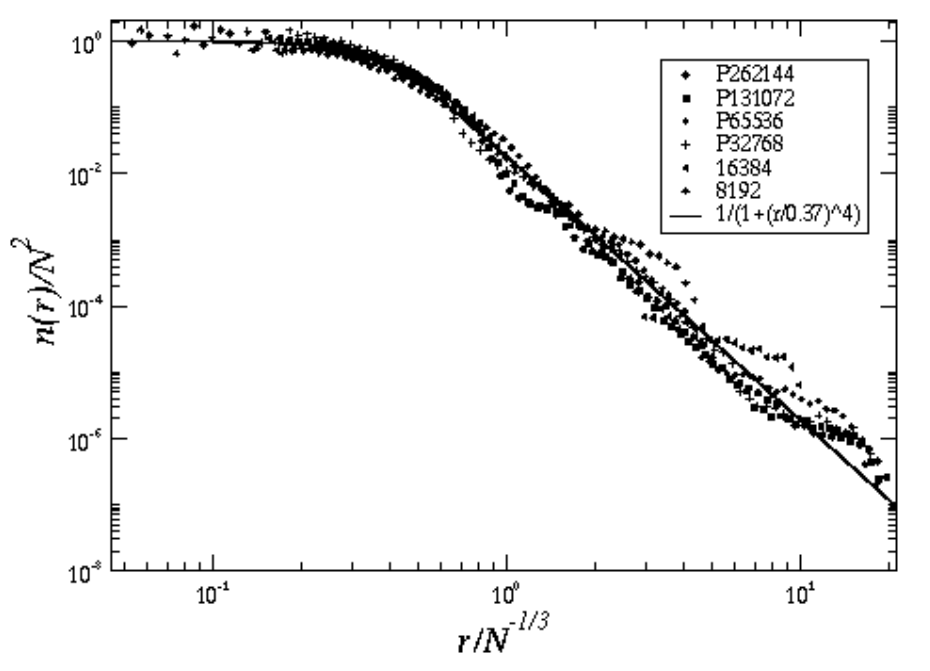


They then “**scatter” off the inner re-expanding core**
With fixed “lagging mass”, the observed scaling can be recovered.

Cold collapse

Properties of virialized “core”

Density profiles (and velocity distributions?) are stable…



Good fit (in agreement with previous literature) given by

$$n(r) = \frac{n_0}{\left(1+\left(\frac{r}{r_0}\right)^4\right)}$$

Cold collapse

Is dynamics collisionless?

Is the dynamics (of any given simulation) collisionless (i.e. mean field like)?

If so, then the final state should be <u>N independent</u>..

This limit is clearly *not* "naïve" $N \rightarrow \infty$.

Amplitude of initial fluctuations in density is evidently crucial
$\rightarrow$extrapolate N **"keeping initial fluctuations fixed"**

"fluctuations fixed" not exact: smallest scales are always N dependent!

But derivation of Vlasov limit: neglect fluctuations <u>below some scale</u>
(see e.g. T. Buchert and A. Dominguez, Astron. Astrophys. 438, 443 (2005))

An extrapolation to test for collisionality

"Recipe":

•In initial condition ($N=N_0$) place a cube of size $\Lambda < R$ around each particle

•Replace each particle by m particles randomly distributed in the cube

[and $N=mN_0 \rightarrow\infty$ when $m \rightarrow\infty$]

Numerical result: final state is (macroscopically) unchanged **if Λ < a**.

Thus: fluctuations below this scale **a** are indeed irrelevant dynamically

[However tests with particles of different masses →
residual (measurable) non-VP effects]

Cold collapse

Some conclusions

•Ejected energy appears to be unbounded above, growing as $N^{1/3}$
(But: have we have reached asymptotic regime? log N dependence of mass…)

•Evolution does appear to be collisionless; defined an appropriate extrapolation
Fluctuations down to interparticle distance appear to be relevant

Cosmological or astrophysical relevance?

<u>In progress:</u> Case with initial velocity

Can show that distinct collisionless limits may be defined!

References

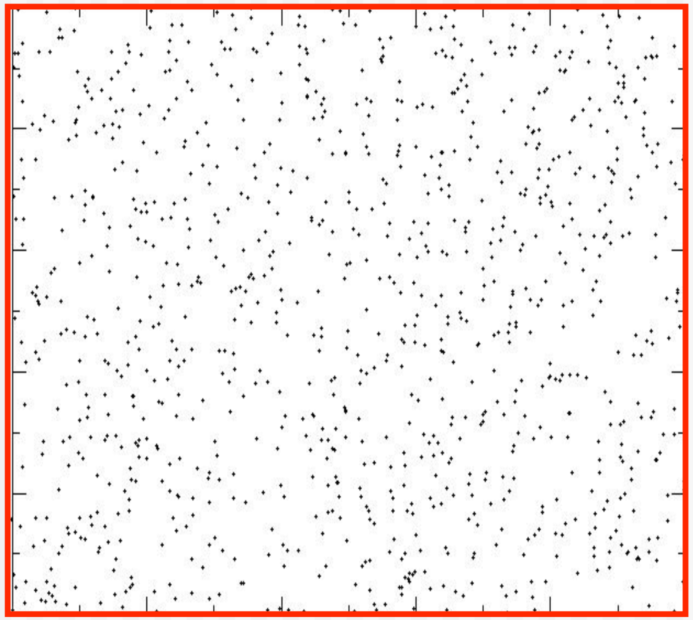
M. Joyce, B .Marcos and F. Sylos Labini
Energy ejection in the collapse of a cold spherical self-gravitating cloud
Mon. Not. R. Aston. Soc., in press (2009)

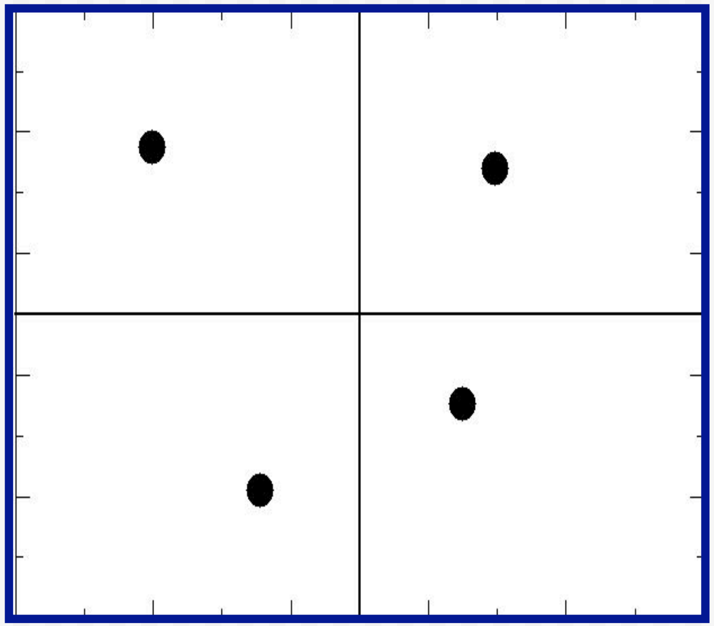
S. Aarseth, D. Lin and D. Papaloizu
On the collapse and violent relaxation of protoglobular clusters
Astroph. J. **324**, 288 (1988)

S. Boily, E. Athanassoula and D. Kroupa, 332, 971 (2002)
Scaling up tides in numerical models of galaxy and halo formation
Mon. Not. R. Aston. Soc., **332**, 971 (2002)

Particle Coarse Grainings

L $\qquad\qquad$ $\ell_{CG} = L/p$





Part I: The problem

Derivation of Vlasov-Poisson equations

see e.g. T. Buchert and A. Dominguez, Astron. Astrophys. 438, 443 (2005)

Exact ("spikey") one particle phase space density $f_K(\mathbf{v},\mathbf{x},t)$

$$f_K(\boldsymbol{x},\boldsymbol{v},t) := \sum_{\alpha=1}^{N} \delta(\boldsymbol{x}-\boldsymbol{x}^{(\alpha)})\,\delta(\boldsymbol{v}-\boldsymbol{v}^{(\alpha)})\ .$$

which obeys the Liouville type equation

$$\frac{\partial f_K}{\partial t} + \boldsymbol{v}\cdot\frac{\partial f_K}{\partial \boldsymbol{x}} + \boldsymbol{g}\cdot\frac{\partial f_K}{\partial \boldsymbol{v}} = 0$$

where

$$\nabla\cdot\boldsymbol{g} :=-\,4\pi G m \int d\boldsymbol{v}\ f_K(\boldsymbol{x},\boldsymbol{v},t)\ ,\ \nabla\times\boldsymbol{g} := \boldsymbol{0}\ .$$

Part I: The problem

Derivation of Vlasov-Poisson equations (2)

see e.g. T. Buchert and A. Dominguez, Astron. Astrophys. 438, 443 (2005)

Now define a <u>smooth</u> one phase space density f(**v**,**x**,t) by <u>coarse-graining</u>, in space and velocity:

$$f(\boldsymbol{x},\boldsymbol{v},t) := \int \frac{d\boldsymbol{x}'}{\mathcal{L}^3}\frac{d\boldsymbol{v}'}{\mathcal{V}^3}\, W\left(\frac{\boldsymbol{x}-\boldsymbol{x}'}{\mathcal{L}}\right) W\left(\frac{\boldsymbol{v}-\boldsymbol{v}'}{\mathcal{V}}\right) f_K(\boldsymbol{x}',\boldsymbol{v}',t)$$

which obeys

$$\frac{\partial f}{\partial t} + \boldsymbol{v}\cdot\frac{\partial f}{\partial \boldsymbol{x}} + \bar{\boldsymbol{g}}\cdot\frac{\partial f}{\partial \boldsymbol{v}} = -\frac{\partial}{\partial \boldsymbol{x}}\cdot\mathbf{S}^{(v)} - \frac{\partial}{\partial \boldsymbol{v}}\cdot\mathbf{S}^{(g)}$$

where

$$\nabla\cdot\bar{\boldsymbol{g}} := -4\pi G m\int d\boldsymbol{v}\, f(\boldsymbol{x},\boldsymbol{v},t)\ ,\ \nabla\times\bar{\boldsymbol{g}} := \mathbf{0}$$

Part I: The problem

Derivation of Vlasov-Poisson equations (3)

see e.g. T. Buchert and A. Dominguez, Astron. Astrophys. 438, 443 (2005)

- Vlasov-Poisson equations obtained neglecting terms on right hand side:

$$\mathbf{S}^{(v)}(\boldsymbol{x}, \boldsymbol{v}, t) := \int \frac{d\boldsymbol{x}'}{\mathcal{L}^3} \frac{d\boldsymbol{v}'}{\mathcal{V}^3} W\left(\frac{\boldsymbol{x}-\boldsymbol{x}'}{\mathcal{L}}\right) W\left(\frac{\boldsymbol{v}-\boldsymbol{v}'}{\mathcal{V}}\right) \times (\boldsymbol{v}'-\boldsymbol{v}) f_K(\boldsymbol{x}', \boldsymbol{v}', t) , \tag{6c}$$

$$\mathbf{S}^{(g)}(\boldsymbol{x}, \boldsymbol{v}, t) := \int \frac{d\boldsymbol{x}'}{\mathcal{L}^3} \frac{d\boldsymbol{v}'}{\mathcal{V}^3} W\left(\frac{\boldsymbol{x}-\boldsymbol{x}'}{\mathcal{L}}\right) W\left(\frac{\boldsymbol{v}-\boldsymbol{v}'}{\mathcal{V}}\right) \times [\boldsymbol{g}(\boldsymbol{x}', t) - \bar{\boldsymbol{g}}(\boldsymbol{x}, t)] f_K(\boldsymbol{x}', \boldsymbol{v}', t). \tag{6d}$$

- V-P: ***decoupling of fluctuations at large scales from those at small scales***

(---> valid above some characteristic length scale)

- CDM has an evident such separation of scales:

discrete "graininess" scales << clustering scales

Part II: Analytical results

Perturbative treatment of the N body problem

M. Joyce, B. Marcos, A. Gabrielli, T. Baertschiger, F. Sylos Labini

Gravitational evolution of a perturbed lattice and its fluid limit, Phys. Rev. Lett. 95:011334(2005)

B. Marcos, T. Baertschiger, M. Joyce, A. Gabrielli, F. Sylos Labini

Linear perturbative theory of the discrete cosmological N body problem, Phys.Rev. D73:103507(2006)

Evolution of exact (discrete) N body system can be solved **perturbatively in displacements** off the lattice

Gives **discrete generalisation** of **Lagrangian perturbative theory** for fluid.

---> Recover the fluid limit and study **N dependent corrections** to it

Part I: finite systems

Cold collapse: theory

$N \rightarrow \infty$ limit: "uniform spherical collapse model" (SCM)

Analytic solution: homologous rescaling

$$r(t) = R(t)r(0)$$

where

$$R(\xi) = \frac{1}{2}(1 + \cos(\xi))$$
$$t(\xi) = \frac{\tau_{scm}}{\pi}(\xi + \sin(\xi))$$

Singularity ($R \rightarrow 0$) at collapse time

$$\tau_{scm} \equiv \sqrt{\frac{3\pi}{32 G \rho_0}}$$

where ρ_0 is the mean mass density ($\rightarrow$ *independent of initial size*)