Non-Gaussianity of the CMB:

imprints at the recombination epoch
and
subsequent non-linear evolution

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Plan of the talk

Primordial non-Gaussianity: a reminder

CMB anisotropies at second-order in perturbation theory

Signal-to-noise ratio and contamination to primordial non-Gaussianity

Based on:
N.B., A. Riotto, JCAP 03, 017 (2009)
D. Nitta, E. Komatsu, N.B., S. Matarrese, A Riotto, JCAP 05, 014 (2009)

N.B, S. Matarrese and A. Riotto, JCAP 06, 02 (2006)
JCAP 01, 19 (2007)

Primordial NG

- Usually parametrized as
  (*e.g. Verde et al 2001; Komatsu & Spergel 2001*)

$$\Phi = \Phi_L + f_{NL} * \left(\Phi_L^2 - \left\langle \Phi_L^2 \right\rangle \right) + \ldots\ldots$$

where **Φ is the large-scale primordial gravitational potential**, Φ_L its linear Gaussian part and **f_{NL}** is the so called **non-linearity parameter**

f_{NL} is model dependent: e.g. standard single-field slow-roll inflation $f_{NL} \sim O(\varepsilon, \eta) << 1$

Acquavia et al. 01; Maldacena 01

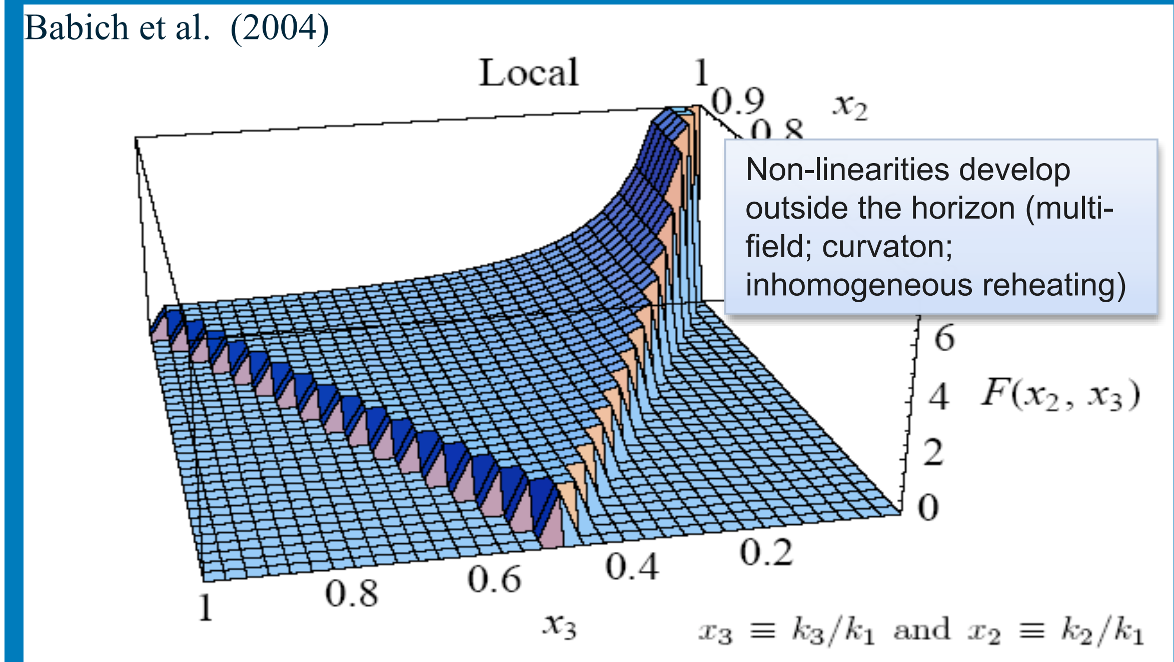
- A non-vanishing **three point function**, or its Fourier transform, the **bispectrum, is an indicator of non-Gaussianity**

$$\langle \Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_2)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)B(k_1, k_2, k_3)$$

$$\left\langle \frac{\Delta T}{T}(n_1)\frac{\Delta T}{T}(n_2)\frac{\Delta T}{T}(n_3)\right\rangle$$

non-Gaussianity shape

(see Babich et al 04; for other shapes see Fergusson and Shellard 08)

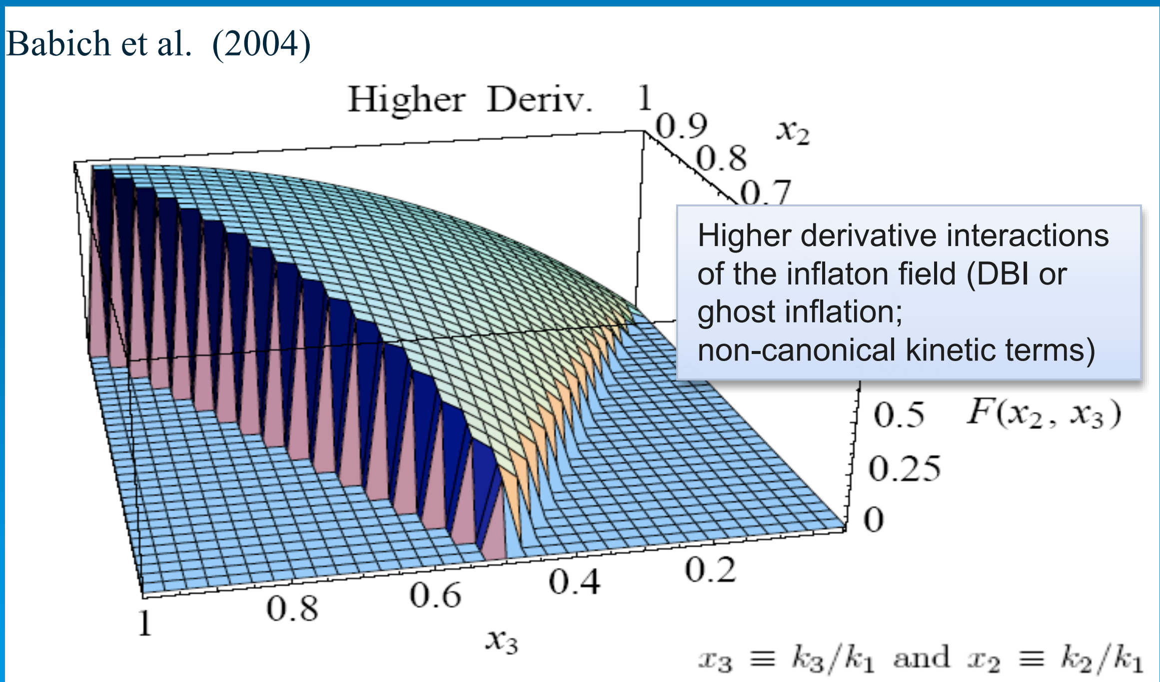


1. Most signal in squeezed triangles: $k_1 << k_2 \sim k_3$

Local Models

$$\Phi(\mathbf{x}) = \Phi_L(\mathbf{x}) + f_{NL}\left(\Phi_L^2(\mathbf{x}) - \langle \Phi_L^2(\mathbf{x})\rangle\right)$$

$-4 < f_{\rm NL}^{\rm loc} < 80$ (95% C.L.)
(Smith et al. 09 from WMAP data)
Planck can reach $f_{\rm NL}^{\rm loc} \sim 5$



2. The bispectrum peaks for equilateral triangles: $k_1 = k_2 = k_3$

NG non-local in real space operators with gradients

$-151 < f_{\rm NL}^{\rm equil} < 253$ (95% C.L.)
(Komatsu et al. 09 from WMAP data)
Planck can reach $f_{\rm NL}^{\rm equil} \sim 67$

Non-primordial sources of NG

✓Any non-linearity can contribute to NG and this can contaminate the extraction of the primordial signal

✓ Need to specify of which primordial non-Gaussianity we study the contamination from the non-primordial sources(local, equilateral..)

✓ We will focus on the non-linearities from Boltzmann equation at second-order for photons, baryons and CDM

In particular: NG from gravity and non-linear dynamics of the photon-baryon fluid at recombination

(many other effects: e.g.

- reionaziation: Cooray and Hu 2000
- ISW-lensing bispectrum, Goldberg & Spergel '99, Smith & Zaldarriaga 06; Serra and Cooray 08; Hanson et al. 09
- lensing-Rees Sciama effect, Mangilli and Verde 09
- Point sources, Babich & Pierpaoli 08
- ISW-Sunyaev-Zel'dovich bispectrum, Goldberg & Spergel `99, Komatsu & Spergel 01, Smith & Zaldarriaga 06)

Second-order CMB Anisotropies

$$\frac{df}{d\eta} = a\,\underbrace{C[f]}_{\text{Collision term}}$$

$$\frac{df}{d\eta} = \underbrace{\frac{\partial f}{\partial \eta} + \frac{\partial f}{\partial x^i}\frac{dx^i}{d\eta} + \frac{\partial f}{\partial p}\frac{dp}{d\eta} + \frac{\partial f}{\partial n^i}\frac{dn^i}{d\eta}}_{\text{Gravity effects}}$$

Metric perturbations: Poisson gauge

$$\boxed{ds^2 = a^2(\eta)\left[-e^{2\Phi}d\eta^2 + 2\omega_i dx^i d\eta + (e^{-2\Psi}\delta_{ij} + \chi_{ij})dx^i dx^j\right]}$$

$$\Phi = \Phi^{(1)} + \frac{1}{2}\Phi^{(2)}, \qquad \psi = \psi^{(1)} + \frac{1}{2}\psi^{(2)}$$

Example: using the geodesic equation for the photons

$$\frac{1}{p}\frac{dp}{d\eta} = -\mathcal{H} + \Psi' - \Phi_{,i}\, n^i e^{\Phi+\Psi} - \omega_i' n^i - \frac{1}{2}\chi_{ij}' n^i n^j \longrightarrow$$

Redshift of the photon
(***Sachs-Wolfe and ISW effects***)

PS: Here the photon momentum is **p=** pn^i with $p^2 = g_{ij} P^i P^j$
($P^\mu = dx^\mu(\lambda)/d\lambda$ quadri-momentum vector)

The 2nd-order photon Boltzmann equation

$$\Delta^{(2)\prime} + n^i \frac{\partial \Delta^{(2)}}{\partial x^i} - \tau' \Delta^{(2)} = S$$

N.B: for a derivation of the Boltzmann equations see also C. Pitrou CQG 09 (includes polarization); Senatore, Tassev, Zaldarriaga, arXiv:0812.36523

$$\Delta = \Delta^{(1)} + \Delta^{(2)}/2$$

$$\Delta^{(2)}(x^i, n^i, \eta) = \frac{\int dpp^3 f^{(2)}}{\int dpp^3 f^{(0)}}$$

with $\tau' = -n_e \sigma_T a$ optical depth

Source term $S = S^{(2)} + S^{(I\times I)}$

Second-order baryon velocity

Sachs-Wolfe effect

$$S^{(2)} = \underline{-\tau'(\Delta^{(2)}_{00} + 4\Phi^{(2)})} + 4(\Phi^{(2)} + \Psi^{(2)})' - 8\omega_i' n^i - 4\chi_{ij}' n^i n^j - \tau' \left[4\mathbf{v}^{(2)} \cdot \mathbf{n} - \frac{1}{2} \sum_{m=-2}^{2} \frac{\sqrt{4\pi}}{5^{3/2}} \Delta^{(2)}_{2m} Y_{2m}(\mathbf{n}) \right]$$

$$\begin{aligned} S^{(I\times I)} &= -8\Delta^{(1)}\left(\Psi^{(1)\prime} - \Phi^{(1)}_{,i} n^i\right) + 2n^i(\Phi^{(1)} + \Psi^{(1)})\partial_i(\Delta^{(1)} + 4\Phi^{(1)}) \\ &+ \left[(\Phi^{(1)}_{,j} + \Psi^{(1)}_{,j}) n^i n^j + (\Phi^{,i} - \Psi^{,i})\right] \frac{\partial \Delta^{(1)}}{\partial n^i} \\ &- \tau' \Bigg[2\delta_e^{(1)}\left(4\mathbf{v}\cdot\mathbf{n} + \Delta_0^{(1)} - \Delta^{(1)} + \frac{1}{2}\Delta_2^{(1)} P_2(\hat{\mathbf{v}}\cdot\mathbf{n})\right) \\ &+ 2(\mathbf{v}\cdot\mathbf{n})\left[\Delta_0^{(1)} + 3\Delta_0^{(1)} - \Delta_2^{(1)}\left(1 - \frac{5}{2}P_2(\hat{\mathbf{v}}\cdot\mathbf{n})\right)\right] - v\Delta_1^{(1)}(5 + 4P_2(\hat{\mathbf{v}}\cdot\mathbf{n})) + 14(\mathbf{v}\cdot\mathbf{n})^2 - 2v^2 \Bigg] \end{aligned}$$

Gravitational lensing

Quadratic-Doppler effect

Coupling velocity and linear photon anisotropies

CMB angular bispectrum

$$B_{l_1 l_2 l_3}^{m_1 m_2 m_3} \equiv \langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle ,$$

$$B_{l_1 l_2 l_3} = \sum_{\text{all } m} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} B_{l_1 l_2 l_3}^{m_1 m_2 m_3},$$

$$\Delta^{(1)} + ik\mu\Delta^{(1)} - \tau'\Delta^{(1)} = S^{(1)}(\mathbf{k}, \hat{\mathbf{n}}, \eta)$$
$$\Delta^{(2)} + ik\mu\Delta^{(2)} - \tau'\Delta^{(2)} = S^{(2)}(\mathbf{k}, \hat{\mathbf{n}}, \eta)$$

$$S_{lm}^{(2)}(\mathbf{k}) = \int \frac{d^3k'}{(2\pi)^3} \int d^3k'' \delta^3(\mathbf{k}' + \mathbf{k}'' - \mathbf{k}) \times \mathcal{S}_{lm}^{(2)}(\mathbf{k}', \mathbf{k}'', \mathbf{k}) \zeta(\mathbf{k}') \zeta(\mathbf{k}'')$$

Harmonic components of the CMB source function

CMB angular bispectrum (II)

$$a_{lm}^{(1)} = 4\pi(-i)^l \int \frac{d^3k}{(2\pi)^3} g_l(k) Y_{lm}^* \zeta(\mathbf{k})$$

$$\Phi(\mathbf{x}) = \Phi_L(\mathbf{x}) + f_{NL}\left(\Phi_L^2(\mathbf{x}) - \langle\Phi_L^2(\mathbf{x})\rangle\right)$$

$$a_{lm}^{(2)} = \frac{4\pi}{8}(-i)^l \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3k'}{(2\pi)^3} \int d^3k'' \delta^3(\mathbf{k}' + \mathbf{k}'' - \mathbf{k}) \times \sum_{l'm'} \underbrace{F_{lm}^{l'm'}(\mathbf{k}', \mathbf{k}'', \mathbf{k})}_{\text{Second-order radiation transfer function}} Y_{l'm'}^*(\hat{\mathbf{k}}) \zeta(\mathbf{k}')\zeta(\mathbf{k}'')$$

Second-order radiation transfer function

Primordial curvature perturbation

$$F_{lm}^{l'm'}(\mathbf{k}', \mathbf{k}'', \mathbf{k}) = i^l \sum_{\lambda\mu} (-1)^m (-i)^{\lambda - l'} \mathcal{G}_{ll'\lambda}^{-mm'\mu} \times \sqrt{\frac{4\pi}{2\lambda+1}} \int_0^{\eta_0} d\eta e^{-\tau} \underbrace{\mathcal{S}_{\lambda\mu}^{(2)}(\mathbf{k}', \mathbf{k}'', \mathbf{k})}_{\text{CMB source function}} j_{l'}[k(\eta - \eta_0)].$$

Nitta, Komatsu, N.B, Matarrese, Riotto 09

CMB source function

A closer look at the CMB source function

Two types of contributions:

1) Intrinsically second-order term $\Theta^{(2)} = (\Delta^{(2)}_{00}/4 + \Phi^{(2)})$

   on large scales it gives rise to the Sachs-Wolfe effect;

   on small scales it grows as η^2

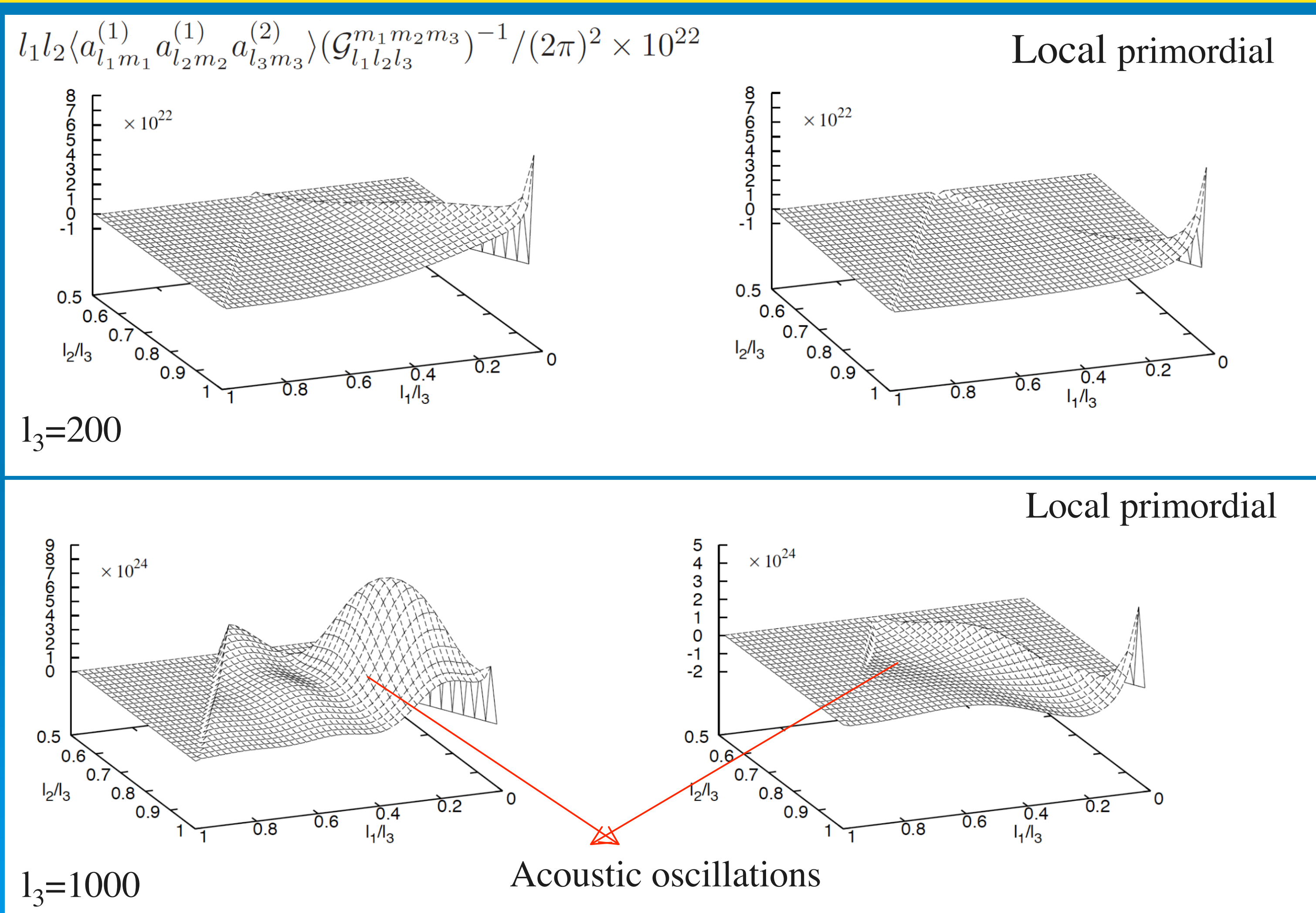
   (first pointed out by Pitrou, Uzan, Bernadeau 08)

2) (first-order)^2-terms (oscillating/constant in time)

Second-order bispectrum from products of 1st-order terms

$$B_{l_1 l_2 l_3} \equiv \sum_{\text{all}\, m} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle$$

Maximum signal in the squeezed triangles, $l_1 \ll l_2 \sim l_3$, similar to the local primordial bispectrum



Shape of the second-order bispectrum from products of 1st-order terms

✓ The bispectrum has the maximum signal in the squeezed triangles $l_1 \ll l_2 \sim l_3$, as the local-type primordial bispectrum: both generate non-linearities via products of first-order terms in position space

✓ However the shapes are sufficiently different

- different dependence on the transfer functions (acoustic oscillations); the primordial bispectrum contain $[g_l(k)]^3$ the 2^{nd}-order one goes like $[g_l(k)]^\alpha$, with $2 \leq \alpha \leq 4$

- second-order effects are not scale-invariant because of extra powers of k (e.g. from velocity terms) $B(k_1,k_2,k_3) \sim (k_1)^m (k_2)^n / (k_1)^3 (k_2)^3$+cycl. still peaks in the squeezed con configuration with $m,n \leq 3$

Signal to Noise ratio

See Spergel and Goldberg '99; Cooray and Hu 200; Komatsu and Spergel 2001

$$\chi^2 \equiv \sum_{2\le l_1 \le l_2 \le l_3} \frac{\left(B^{obs}_{l_1 l_2 l_3} - \sum_i A_i B^{(i)}_{l_1 l_2 l_3}\right)^2}{\sigma^2_{l_1 l_2 l_3}}$$

$$\sigma^2_{l_1 l_2 l_3} \equiv \langle B^2_{l_1 l_2 l_3} \rangle - \langle B_{l_1 l_2 l_3} \rangle^2 \approx \mathcal{C}_{l_1} \mathcal{C}_{l_2} \mathcal{C}_{l_3} \Delta_{l_1 l_2 l_3}$$

$B^{(i)}_{l_1 l_2 l_3}$: primordial or secondary bispectra

Fisher matrix

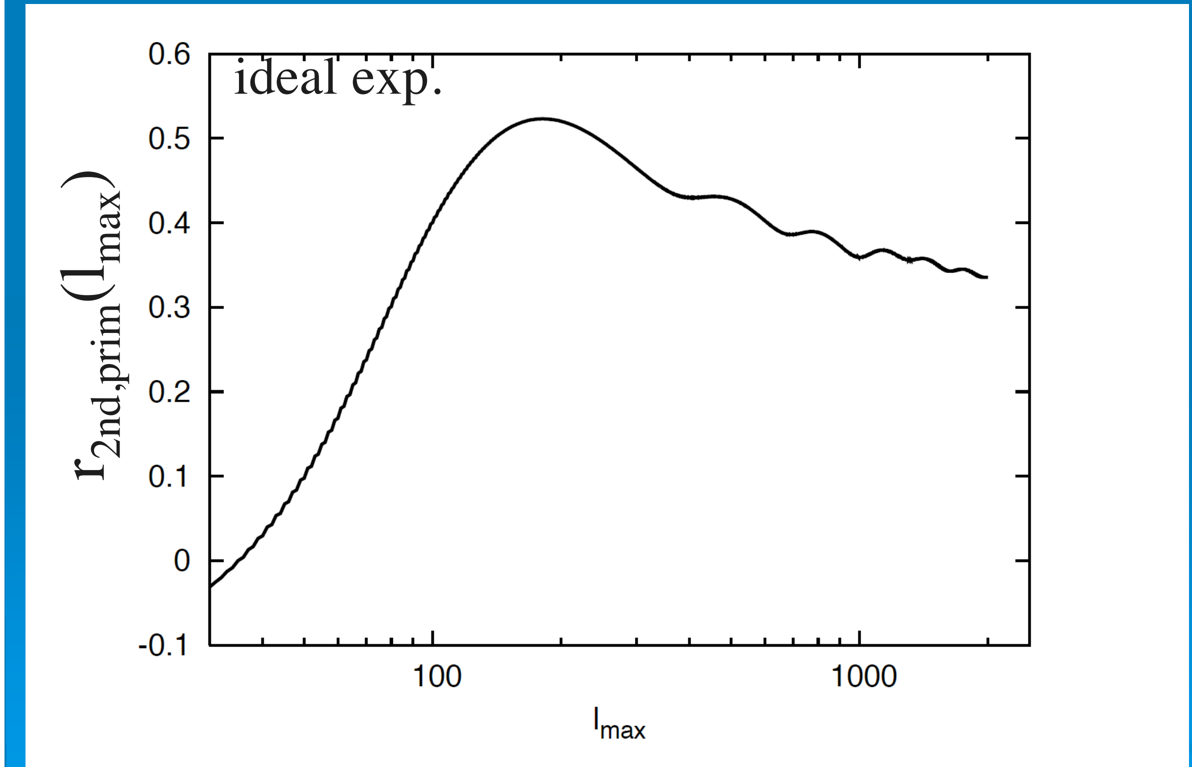
$$\boxed{F_{ij} \equiv \sum_{2\le l_1 \le l_2 \le l_3} \frac{B^{(i)}_{l_1 l_2 l_3} B^{(j)}_{l_1 l_2 l_3}}{\sigma^2_{l_1 l_2 l_3}}}$$

$$\boxed{\left(\frac{S}{N}\right)_i \equiv \frac{1}{\sqrt{F^{-1}_{ii}}},}$$

Cross-correlation

$$r_{ij} = \frac{F_{ij}^{-1}}{\sqrt{F_{ii}F_{ij}}}$$

How similar are 2 bispectra?

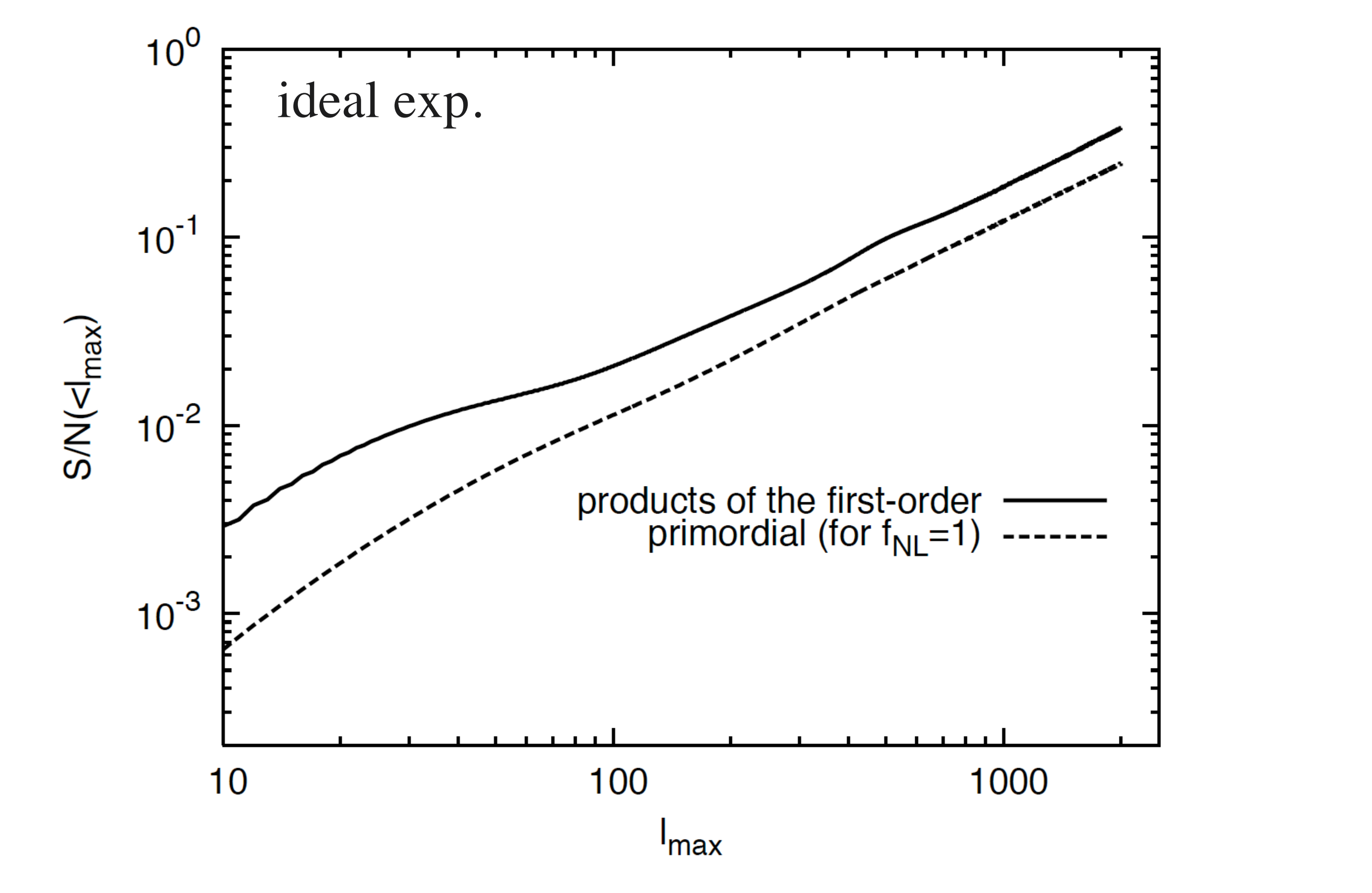


2^{nd}-order bispectrum and local primordial are fairly similar

$r_{2nd,prim} \sim 0.5$ at lmax~200

$r_{2nd,prim} \sim 0.3$ at lmax~2000

Signal to Noise ratio: numerical results for (first-order)^2-terms



(Nitta, Komatsu, N.B., Matarrese, Riotto, JCAP 09)

(S/N) from the (first-order×first-order) terms is about 0.4 at l_{max}≈2000 for an ideal full-sky experiment

Contamination to primordial f_NL of the local type :

We fit the primordial bispectrum template to th 2nd-order bispectrum:
define the best-fitting 'contamination' f^{cont}_{NL} by minimizing

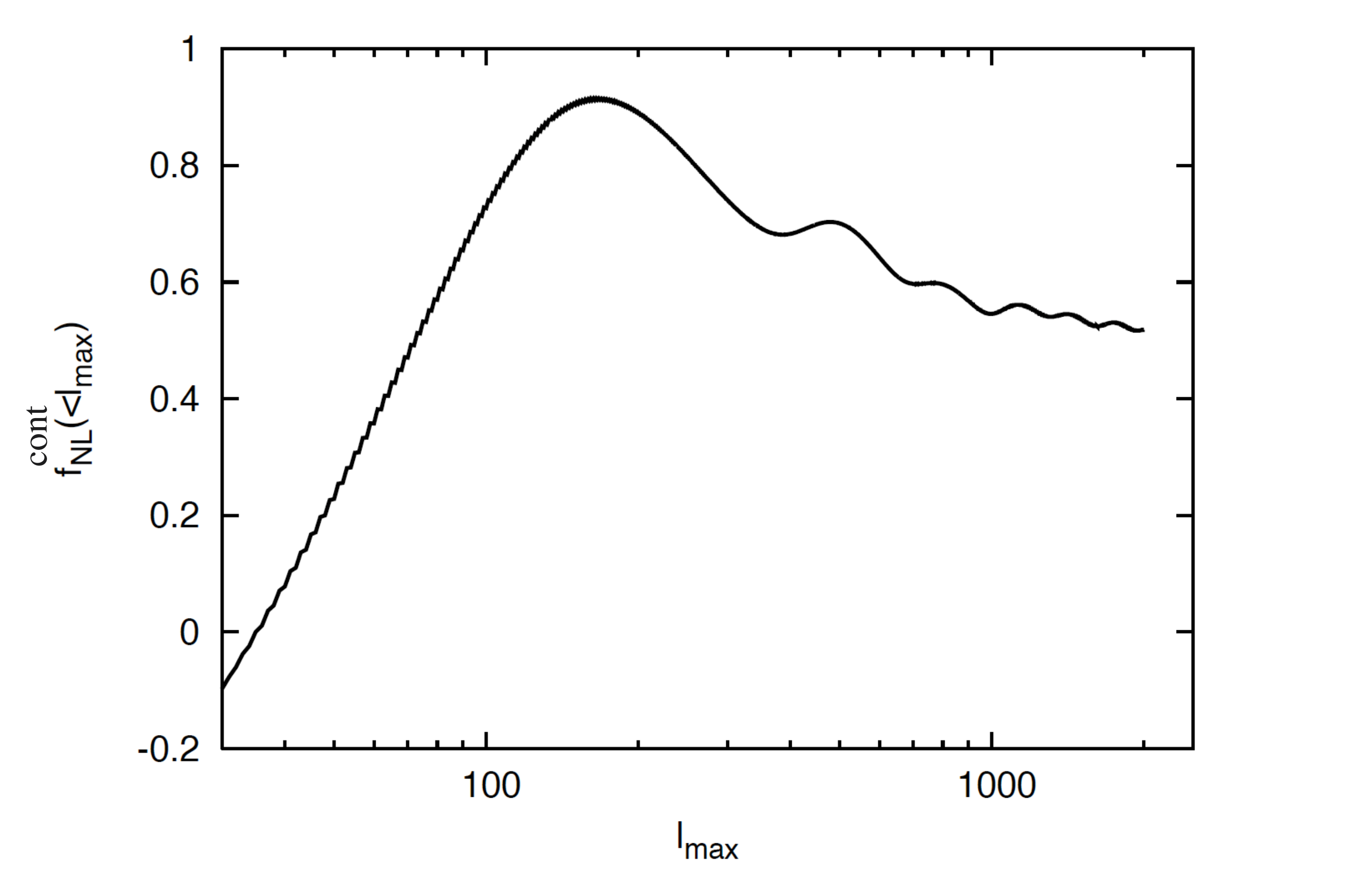
$$\chi^2 = \sum_{2\leq l_1\leq l_2\leq l_3} \frac{\left(f^{\mathrm{cont}}_{NL} B^{prim}_{l_1 l_2 l_3} - B^{2nd}_{l_1 l_2 l_3}\right)^2}{\sigma^2_{l_1 l_2 l_3}}$$

$$\downarrow$$

$$\boxed{f^{\mathrm{cont}}_{NL} = \left.\frac{F_{2nd,prim}}{F_{prim,prim}}\right|_{f_{NL}=1} = \frac{1}{N} \sum_{2\leq l_1\leq l_2\leq l_3} \frac{B^{2nd}_{l_1 l_2 l_3} B^{prim}_{l_1 l_2 l_3}}{\sigma^2_{l_1 l_2 l_3}}}$$

$$N = \sum_{2\leq l_1\leq l_2\leq l_3} \frac{{B^{prim}_{l_1 l_2 l_3}}^2}{\sigma^2_{l_1 l_2 l_3}}$$

Contamination to primordial f_NL of the local type from (first-order)^2-terms



(Nitta, Komatsu, N.B., Matarrese, Riotto, JCAP 09)

Contamination is 0.9 at l_{max}≈200 and 0.5 at l_{max}≈2000 vs 5 which is the minimum detectable value for Planck

Non-linear dynamics at recombination

On small scales, i.e. modes k >> k_{eq}, the second-order anisotropies at recombination are dominated by the 2nd-oder gravitational potential sourced by dark matter perturbations

$$\Phi^{(2)} \simeq \Psi^{(2)} = \Psi^{(2)}(0) - \frac{1}{14}\left(\partial_k \Phi^{(1)} \partial^k \Phi^{(1)} - \frac{10}{3}\frac{\partial_i \partial^j}{\nabla^2}\left(\partial_i \Phi^{(1)} \partial_j \Phi^{(1)}\right)\right)\eta^2$$

Initial conditions that contain the primordial NG

in Fourier space gives the convolution kernel

$$\boxed{G(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k})\eta^2 = \left[\mathbf{k}_1 \bullet \mathbf{k}_2 - \frac{10}{3}\frac{(\mathbf{k}\bullet\mathbf{k}_2)(\mathbf{k}\bullet\mathbf{k}_1)}{k^2}\right]\eta^2}$$

As a generalization to the well known expression at linear-order

$$\Theta^{(2)} = \frac{1}{4}\Delta_{00}^{(2)} + \Phi^{(2)} \sim A\cos[kc_s\eta]\, e^{-(k/k_D)^2} - R\Phi^{(2)} + S$$

(For details see Pitrou et al. 08; see also N.B, Matarrese, Riotto 07)

On small scales the combination of the damping effects AND the growth of the potential as η^2 make dominant the term

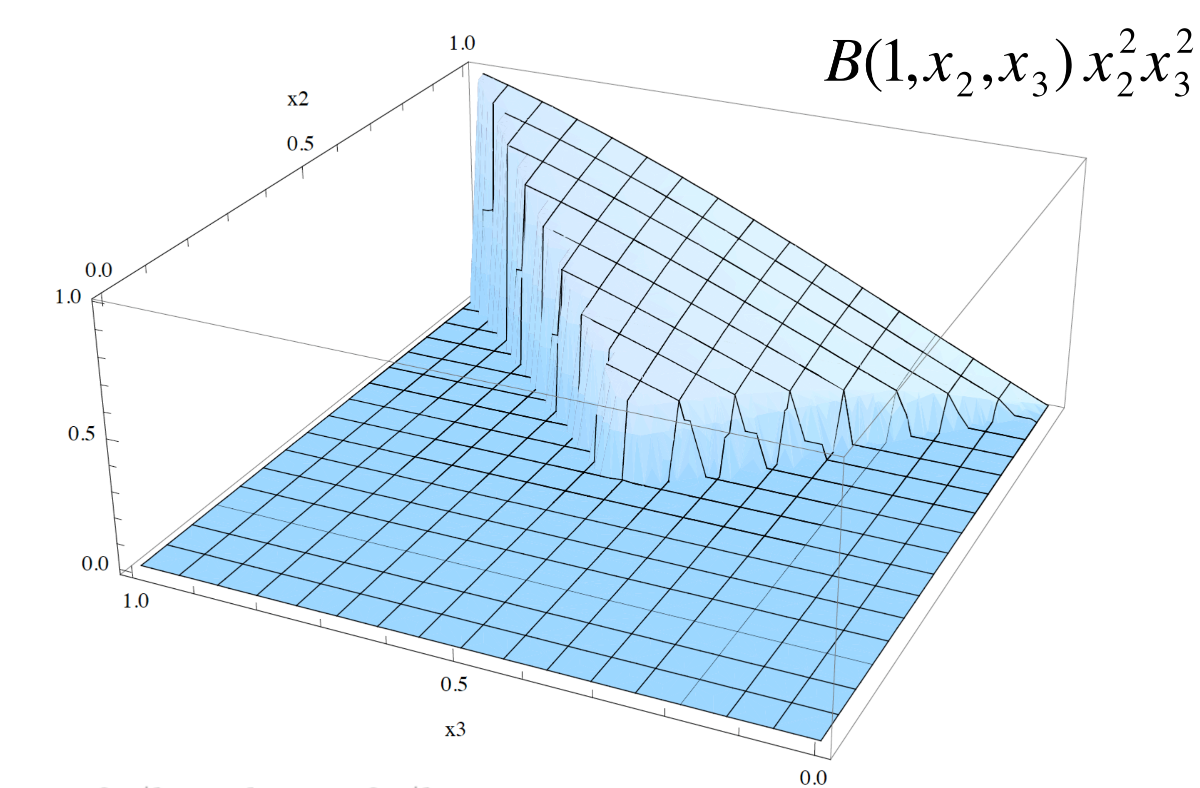
$$-R\Phi^{(2)} = -\frac{R}{14} G(\mathrm{k}_1,\mathrm{k}_2,\mathrm{k})\eta^2 \Phi^{(1)}(\mathrm{k}_1)\Phi^{(1)}(\mathrm{k}_2)$$

Non-Gaussianity from 2nd-order gravitational potential

$$\Theta^{(2)} \cong -R\Phi^{(2)} \cong -\frac{R}{14} G(\mathrm{k}_1,\mathrm{k}_2,\mathrm{k})\eta^2 \Phi^{(1)}(\mathrm{k}_1)\Phi^{(1)}(\mathrm{k}_2)$$

This effect is a causal one, i.e. developing on small scales; its origin is gravitational (due to the non-linear growth sourced by dark matter perturbations)

We expect the corresponding CMB bispectrum will be of the equilateral type.



$x_3 \equiv k_3/k_1$ and $x_2 \equiv k_2/k_1$

Important comment:
so, even if the NG at recombination is dominated by this effect, it will have a minimal contamination to the local primordial NG

Correlation to primordial f_{NL} of the intrinsically second-order term $\theta^{(2)}=-R\Phi^{(2)}$

Fisher matrix

$$F_{ij} = \frac{f_{\rm sky}}{\pi} \frac{1}{(2\pi)^2} \int d^2\ell_1 d^2\ell_2 d^2\ell_3 \, \delta^{(2)}(\vec{\ell}_{123}) \, \frac{B^i(\ell_1,\ell_2,\ell_3)\, B^j(\ell_1,\ell_2,\ell_3)}{6\, C(\ell_1)\, C(\ell_2)\, C(\ell_3)}$$

For EQUILATERAL primordial f^{eq}_{NL}

$$\left(\frac{S}{N}\right)_{\rm equil} = \frac{1}{\sqrt{F^{-1}_{\rm equil,equil}}} \simeq 12.6 \times 10^{-3} f_{\rm NL}^{\rm equil}$$

$$\left(\frac{S}{N}\right)_{\rm rec} = \frac{1}{\sqrt{F^{-1}_{\rm rec,rec}}} \simeq 0.1$$

$$r_{\rm rec,equil} = \frac{F^{-1}_{\rm rec,equil}}{\sqrt{F^{-1}_{\rm equil,equil} F^{-1}_{\rm rec,rec}}} \simeq -0.53$$

$$d_{\rm rec} = F_{\rm rec,rec} F^{-1}_{\rm rec,rec} \simeq 1.4$$

$$d_{\rm equil} = F_{\rm equil,equil} F^{-1}_{\rm equil,equil} \simeq 1.4$$

As a confirmation of our expectations the NG from recombination (governed by the non-linear evolution of the 2nd-order gravitational potential) shows a quite high correlation with an equilateral primordial bispectrum

N.B, Riotto JCAP 09

✓ Degradation of the 1-σ uncertainty of f^{eq}_{NL}

$r_{rec,equil} = -0.53$ translates into an increase of the minimum detectable value for f^{eq}_{NL}. We find a mimimum value of

$$\boxed{f_{\rm NL}^{\rm equil} \simeq 79} \longleftrightarrow \boxed{\Delta f_{\rm NL}^{\rm equil} = \mathcal{O}(10)}$$

Recall: f^{eq}_{NL}=67, not accounting for the cross-correlation (i.e. not marginalized over the signal from recombination). It corresponds to a shift of O(10)

✓ Contamination to primordial f_{NL}

Contamination to equilateral	$f_{NL}^{cont} = O(10)$
Contamination to local	$f_{NL}^{cont} \approx 0.3$

Given the 1-σ uncertainty $f^{loc}_{NL} \sim 5$ for local
$f^{eq}_{NL} \sim 67$ for equilateral these numbers are not relevant.

So the contamination from the intrinsically second-order term $\theta^{(2)} = -R\Phi^{(2)}$ to a <u>primordial local NG</u> is minimal;
what is relevant is the degradation in the minimum detectable value of f^{eq}_{NL} for <u>primordial equilateral NG</u>

A check for our model:
S/N for the primordial equilateral bispectrum

Use the flat sky approximation

$$a(\vec{\ell}) = \int \frac{dk^z}{2\pi} e^{ik^z(\eta_0-\eta_r)} \Phi(\mathbf{k}') \tilde{\Delta}^T(\ell, k^z)$$

transfer function on small scales
($l >> l_* \sim 750$; $a \sim 3$)

$$\tilde{\Delta}^T(\ell, k^z) = a\,(\eta_0 - \eta_r)^{-2} e^{-1/2(\ell/\ell_*)^{1.2}} e^{-1/2(|k_z|/k_*)^{1.2}}$$

power spectrum
(see also Babich & Zaldarriaga 04)

$$C(\ell) \simeq a^2 \frac{A}{\pi} \frac{\ell_*}{\ell^3}\, e^{-(\ell/\ell_*)^{1.2}}$$

Bispectrum

$$B_{\rm equil}(\ell_1,\ell_2,\ell_3) = \frac{(\eta_0-\eta_r)^2}{(2\pi)^2} \int dk_1^z dk_2^z dk_3^z \delta^{(1)}(k_{123}^z) B_{\rm equil}(k_1', k_2', k_3') \tilde{\Delta}^T(\ell_1, k_1^z) \tilde{\Delta}^T(\ell_2, k_2^z) \tilde{\Delta}^T(\ell_3, k_3^z) \quad (5$$

$$B_{\rm equil}(k_1,k_2,k_3) = f_{\rm NL}^{\rm equil} \cdot 6A^2 \cdot \left(-\frac{1}{k_1^3 k_2^3} - \frac{1}{k_1^3 k_3^3} - \frac{1}{k_2^3 k_3^3} - \frac{2}{k_1^2 k_2^2 k_3^2} + \frac{1}{k_1 k_2^2 k_3^3} + (5 \text{ perm.}) \right)$$

$$B_{\rm equil}(\ell_1,\ell_2,\ell_3) = \frac{24 f_1}{(2\pi)^2} f_{\rm NL}^{\rm equil} a^3\, A^2\, e^{-\ell_T^{1.2}/2\ell_*^{1.2}}\, \ell_*^2 \left(-\frac{1}{\ell_1^3 \ell_2^3} - \frac{1}{\ell_1^3 \ell_3^3} - \frac{1}{\ell_2^3 \ell_3^3} - \frac{2}{\ell_1^2 \ell_2^2 \ell_3^2} + \frac{1}{\ell_1 \ell_2^2 \ell_3^3} + (5 \text{ perm.}) \right)$$

S/N for the primordial equilateral bispectrum

Signal-to-noise ratio

$$\left(\frac{S}{N}\right)^2 = \frac{f_{\rm sky}}{\pi}\frac{1}{(2\pi)^2}\int d^2\ell_1 d^2\ell_2 d^2\ell_3\, \delta^{(2)}(\vec{\ell}_{123})\, \frac{B^2_{\rm equil}(\ell_1,\ell_2,\ell_3)}{6\, C(\ell_1)\, C(\ell_2)\, C(\ell_3)}$$

$$\boxed{\left(\frac{S}{N}\right)^2_{\rm equil} \simeq 8\, f_{\rm sky}\, A\, (f_{\rm NL}^{\rm equil})^2\, \ell_{\rm max}}$$

N.B. & Riotto JCAP 09

For f_{sky}=0.8 and l_{max}=2000, for an experiment like Planck gives a minimum detectable

$$\boxed{f_{\rm NL}^{\rm equil} \simeq 66}$$

very good agreement with the numerical results of Smith and Zaldarriaga 06, and Liguori 09

N.B. The S/N scale as $(l_{max})^{1/2}$, not as l_{max} as in the local case;

$(S/N)^2$ receive contribution from the configuration which is peaked at l_1~l_2~l_3, thus it can be written as $\int d^2 l_1\, d^2 l_2\, \delta(l_1 - l_2)/l_1^2 \propto l_{\rm max}$

Non-Gaussianity from 2nd-order gravitational potential

We have estimated in a semi-analytical way the contribution to the CMB anisotropies coming from

$$\Theta^{(2)} \cong -R\Phi^{(2)} \cong -\frac{R}{14} G(\mathrm{k}_1,\mathrm{k}_2,\mathrm{k})\eta^2\Phi^{(1)}(\mathrm{k}_1)\Phi^{(1)}(\mathrm{k}_2)$$

$$G(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}) = \left[\mathbf{k}_1 \bullet \mathbf{k}_2 - \frac{10}{3}\frac{(\mathbf{k}\bullet\mathbf{k}_2)(\mathbf{k}\bullet\mathbf{k}_1)}{k^2}\right]$$

Bispectrum: in the flat-sky approx.

$$\begin{aligned} B_{\rm rec}(\ell_1,\ell_2,\ell_3) &= -\frac{4f_2}{(2\pi)^2}\frac{R}{14}A^2a^2T_0^2(k_{\rm eq}\eta_r)^2\ell_{\rm eq}^2\ell_*^2 e^{-1/2(\ell_1/\ell_*)^{1.2}}e^{-1/2(\ell_2/\ell_*)^{1.2}} \\ &\times \frac{1}{\ell_1^5\ell_2^5}\left[\vec{\ell}_1\cdot\vec{\ell}_2 - \frac{10}{3}\frac{(\vec{\ell}_3\cdot\vec{\ell}_1)(\vec{\ell}_3\cdot\vec{\ell}_2)}{\ell_3^2}\right] + \text{cyclic}\,. \end{aligned}$$

- A: amplitude of the power spectrum of Φ
- $f_2 \approx f_1 = 0.7$
- $R = 3\varrho_b/3\varrho_\gamma \approx 0.3$ at recombination;
- $l_{eq} \approx 160$

- Matter transfer function

$$\Phi^{(1)}(\mathbf{k}) = T(k)\Phi_0(\mathbf{k})$$

$$T(k) \approx 12(k_{\rm eq}/k)^2 \ln[k/8k_{\rm eq}] \equiv T_0(k)\ln[k/8k_{\rm eq}]$$

$T_0 \approx 11$ for the scales of interest ($750 < l_{max} \sim 2000$)

Conclusions

√ We are assessing precisely the level of NG coming from second-order perturbations and their contamination to the primordial NG.

√ Analytical solutions and numerical evaluation in progress
-the (first-order)^2-terms analyzed have low S/N and **minimal contamination to f_NL local for an experiment like Planck**
- **NG from 2nd-order gravitational potential sourced by dark matte at recombination affects the equilateral primordial bispectrum increasing the minimum detectable value by Δf^{EQ}_{NL}=O(10).**
CAUTION: some (first-order)^2-terms and some intrinsically second-order terms still to be studied

√ Future techniques exploiting the CMB polarization and additional independent statistical estimators might help NG detection down to f_{NL}~3: need to compute exactly the ***predicted amplitude and shape of NG from the post-inflationary evolution of perturbations***.